CHAPTER 2

Radical Functions

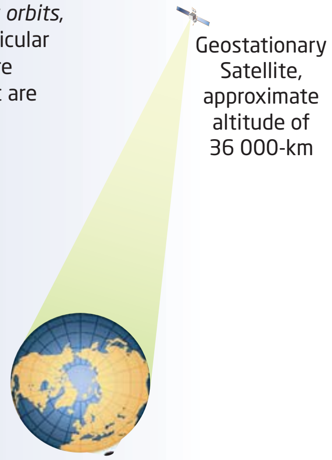
How far can you see from the top of a hill? What range of vision does a submarine's periscope have? How much fertilizer is required for a particular crop? How much of Earth's surface can a satellite "see"? You can model each of these situations using a radical function. The functions can range from simple square root functions to more complex radical functions of higher orders.

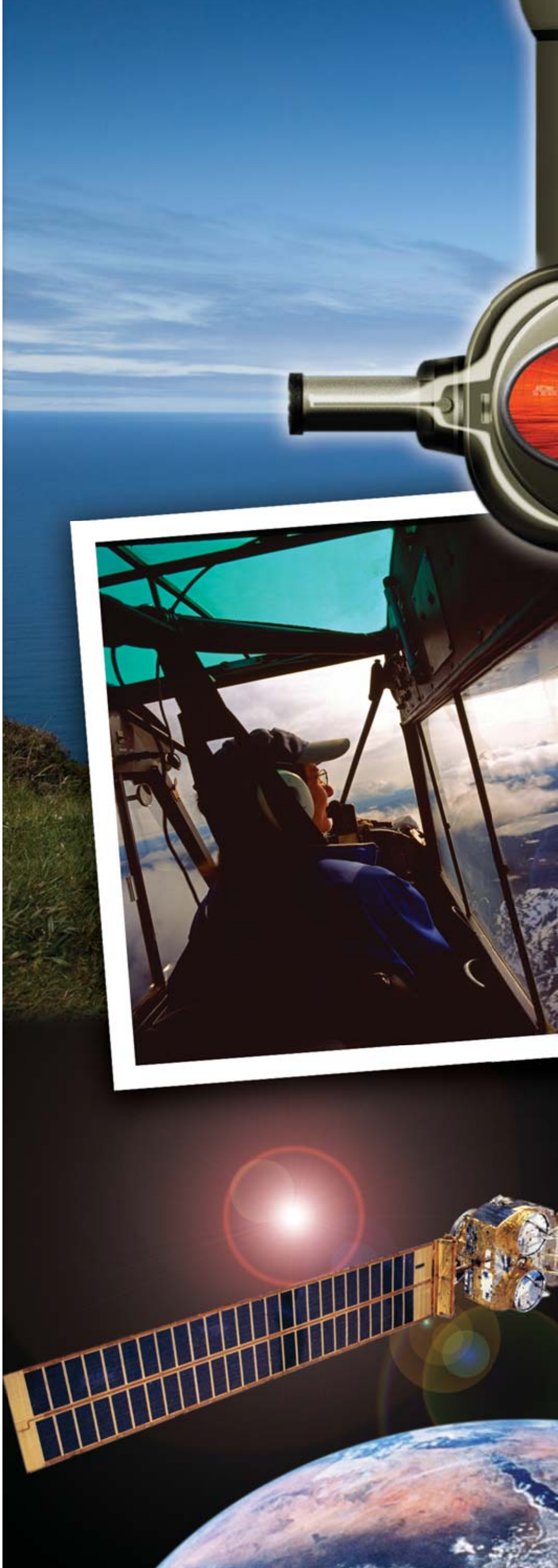
In this chapter, you will explore a variety of square root functions and work with radical functions used by an aerospace engineer when relating the distance to the horizon for a satellite above Earth. Would you expect this to be a simple or a complex radical function?

Did You Know?

Some satellites are put into *polar orbits*, where they follow paths perpendicular to the equator. Other satellites are put into *geostationary orbits* that are parallel to the equator.

Polar orbiting satellites are useful for taking high-resolution photographs. Geostationary satellites allow for weather monitoring and communications for a specific country or continent.





Key Terms

radical function

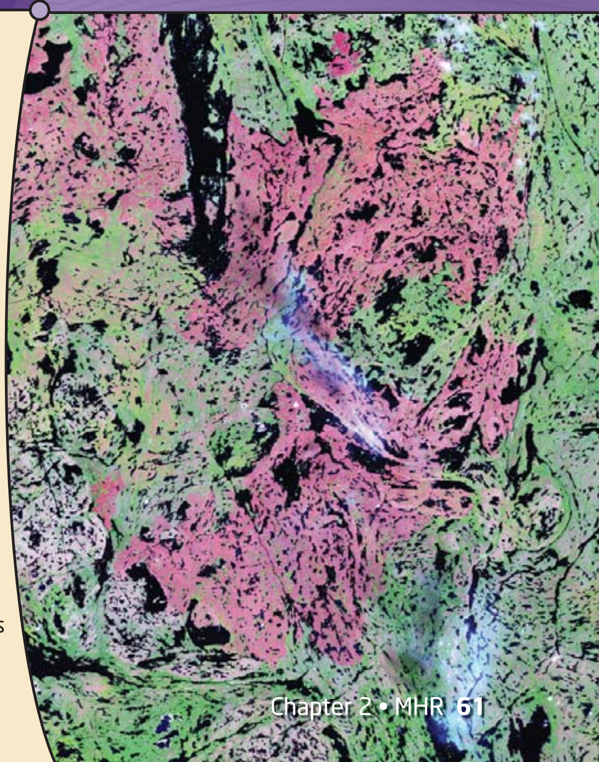
square root of a function

Career Link

Scientists and engineers use remote sensing to create satellite images. They use instruments and satellites to produce information that is used to manage resources, investigate environmental issues, and produce sophisticated maps.

Web Link

To learn more about a career or educational opportunities involving remote sensing, go to www.mcgrawhill.ca/school/learningcentres and follow the links.



Yellowknife Wetlands

2.1

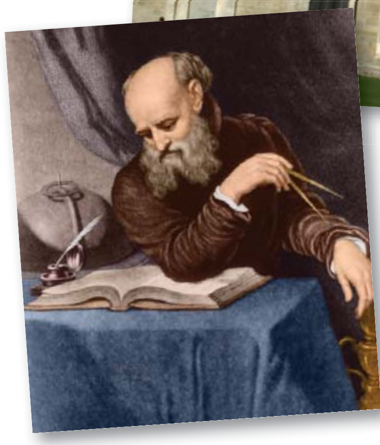
Radical Functions and Transformations

Focus on...

- investigating the function $y = \sqrt{x}$ using a table of values and a graph
- graphing radical functions using transformations
- identifying the domain and range of radical functions

Does a feather fall more slowly than a rock? Galileo Galilei, a mathematician and scientist, pondered this question more than 400 years ago. He theorized that the rate of falling objects depends on air resistance, not on mass. It is believed that he tested his idea by dropping spheres of different masses but the same diameter from the top of the Leaning Tower of Pisa in what is now Italy. The result was exactly as he predicted—they fell at the same rate.

In 1971, during the Apollo 15 lunar landing, Commander David Scott performed a similar demonstration on live television. Because the surface of the moon is essentially a vacuum, a hammer and a feather fell at the same rate.



Web Link

For more information about Galileo or the Apollo 15 mission, go to www.mcgrawhill.ca/school/learningcentres and follow the links.

Investigate a Radical Function

Materials

- grid paper
- graphing technology (optional)

For objects falling near the surface of Earth, the function $d = 5t^2$ approximately models the time, t, in seconds, for an object to fall a distance, d, in metres, if the resistance caused by air can be ignored.

1. a) Identify any restrictions on the domain of this function. Why are these restrictions necessary? What is the range of the function?

b) Create a table of values and a graph showing the distance fallen as a function of time.

2. Express time in terms of distance for the distance-time function from step 1. Represent the new function graphically and using a table of values.

3. For each representation, how is the equation of the new function from step 2 related to the original function?

Reflect and Respond

4. **a)** The original function is a distance-time function. What would you call the new function? Under what circumstances would you use each function?

   b) What is the shape of the graph of the original function? Describe the shape of the graph of the new function.

Link the Ideas

The function that gives the predicted fall time for an object under the influence of gravity is an example of a **radical function**. Radical functions have restricted domains if the index of the radical is an even number. Like many types of functions, you can represent radical functions in a variety of ways, including tables, graphs, and equations. You can create graphs of radical functions using tables of values or technology, or by transforming the base radical function, $y = \sqrt{x}$.

radical function

- a function that involves a radical with a variable in the radicand
- $y = \sqrt{3x}$ and $y = 4\sqrt[3]{5 + x}$ are radical functions.

Example 1

Graph Radical Functions Using Tables of Values

Use a table of values to sketch the graph of each function. Then, state the domain and range of each function.

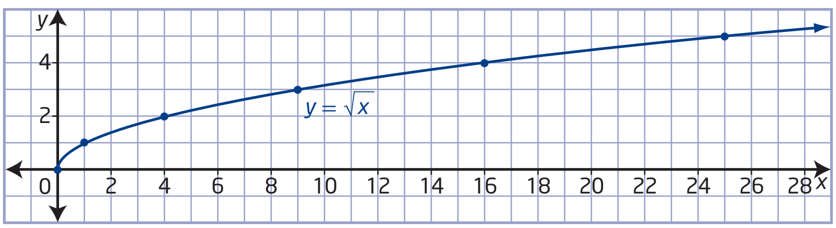
a) $y = \sqrt{x}$ **b)** $y = \sqrt{x - 2}$ **c)** $y = \sqrt{x} - 3$

Solution

a) For the function $y = \sqrt{x}$, the radicand x must be greater than or equal to zero, $x \geq 0$.

x	y
0	0
1	1
4	2
9	3
16	4
25	5

How can you choose values of x that allow you to complete the table without using a calculator?



The graph has an endpoint at (0, 0) and continues up and to the right. The domain is $\{x \mid x \geq 0, x \in \mathrm{R}\}$. The range is $\{y \mid y \geq 0, y \in \mathrm{R}\}$.

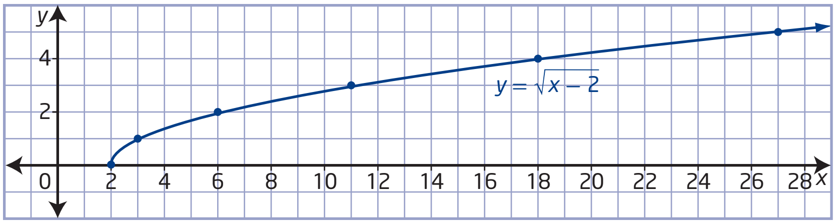
b) For the function $y = \sqrt{x - 2}$, the value of the radicand must be greater than or equal to zero.

$$x - 2 \geq 0$$
$$x \geq 2$$

x	y
2	0
3	1
6	2
11	3
18	4
27	5

How is this table related to the table for $y = \sqrt{x}$ in part a)?

How does the graph of $y = \sqrt{x - 2}$ compare to the graph of $y = \sqrt{x}$?

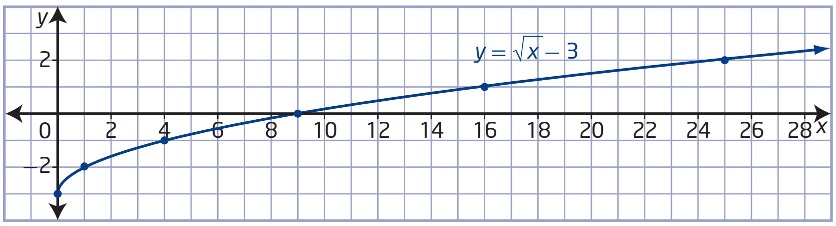


The domain is $\{x \mid x \geq 2, x \in \mathrm{R}\}$. The range is $\{y \mid y \geq 0, y \in \mathrm{R}\}$.

c) The radicand of $y = \sqrt{x} - 3$ must be non-negative.
$x \geq 0$

x	y
0	−3
1	−2
4	−1
9	0
16	1
25	2

How does the graph of $y = \sqrt{x} - 3$ compare to the graph of $y = \sqrt{x}$?



The domain is $\{x \mid x \geq 0, x \in \mathrm{R}\}$ and the range is $\{y \mid y \geq -3, y \in \mathrm{R}\}$.

Your Turn

Sketch the graph of the function $y = \sqrt{x + 5}$ using a table of values. State the domain and the range.

Graphing Radical Functions Using Transformations

You can graph a radical function of the form $y = a\sqrt{b(x - h)} + k$ by transforming the graph of $y = \sqrt{x}$ based on the values of a, b, h, and k. The effects of changing parameters in radical functions are the same as the effects of changing parameters in other types of functions.

- Parameter a results in a vertical stretch of the graph of $y = \sqrt{x}$ by a factor of $|a|$. If $a < 0$, the graph of $y = \sqrt{x}$ is reflected in the x-axis.
- Parameter b results in a horizontal stretch of the graph of $y = \sqrt{x}$ by a factor of $\frac{1}{|b|}$. If $b < 0$, the graph of $y = \sqrt{x}$ is reflected in the y-axis.
- Parameter h determines the horizontal translation. If $h > 0$, the graph of $y = \sqrt{x}$ is translated to the right h units. If $h < 0$, the graph is translated to the left $|h|$ units.
- Parameter k determines the vertical translation. If $k > 0$, the graph of $y = \sqrt{x}$ is translated up k units. If $k < 0$, the graph is translated down $|k|$ units.

Example 2

Graph Radical Functions Using Transformations

Sketch the graph of each function using transformations. Compare the domain and range to those of $y = \sqrt{x}$ and identify any changes.

a) $y = 3\sqrt{-(x - 1)}$ **b)** $y - 3 = -\sqrt{2x}$

Solution

a) The function $y = 3\sqrt{-(x - 1)}$ is expressed in the form $y = a\sqrt{b(x - h)} + k$. Identify the value of each parameter and how it will transform the graph of $y = \sqrt{x}$.

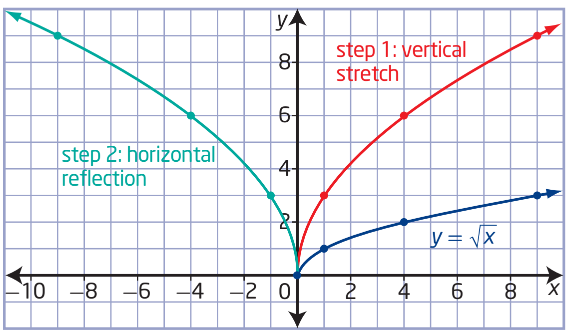
Why is it acceptable to have a negative sign under a square root sign?

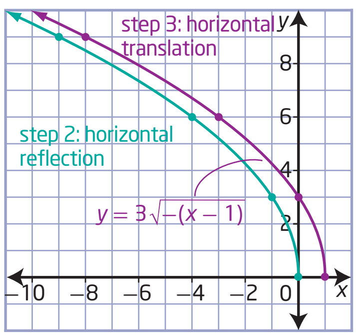
- $a = 3$ results in a vertical stretch by a factor of 3 (step 1).
- $b = -1$ results in a reflection in the y-axis (step 2).
- $h = 1$ results in a horizontal translation of 1 unit to the right (step 3).
- $k = 0$, so the graph has no vertical translation.

Method 1: Transform the Graph Directly

Start with a sketch of $y = \sqrt{x}$ and apply the transformations one at a time.

In what order do transformations need to be performed?





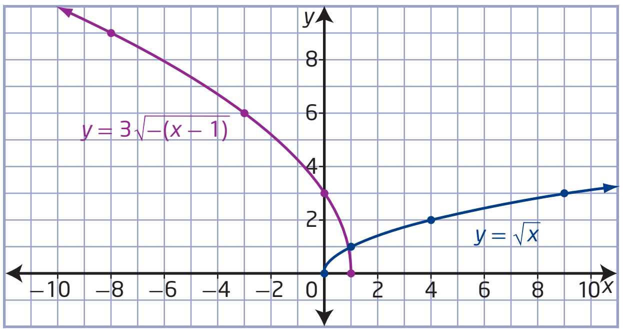
Method 2: Map Individual Points

Choose key points on the graph of $y = \sqrt{x}$ and map them for each transformation.

How can you use mapping notation to express each transformation step?

Transformation of $y = \sqrt{x}$	Mapping
Vertical stretch by a factor of 3	$(0, 0) \to (0, 0)$ $(1, 1) \to (1, 3)$ $(4, 2) \to (4, 6)$ $(9, 3) \to (9, 9)$
Horizontal reflection in the y-axis	$(0, 0) \to (0, 0)$ $(1, 3) \to (-1, 3)$ $(4, 6) \to (-4, 6)$ $(9, 9) \to (-9, 9)$
Horizontal translation of 1 unit to the right	$(0, 0) \to (1, 0)$ $(-1, 3) \to (0, 3)$ $(-4, 6) \to (-3, 6)$ $(-9, 9) \to (-8, 9)$

Plot the image points to create the transformed graph.



The function $y = \sqrt{x}$ is reflected horizontally, stretched vertically by a factor of 3, and then translated 1 unit right. So, the graph of $y = 3\sqrt{-(x - 1)}$ extends to the left from $x = 1$ and its domain is $\{x \mid x \leq 1, x \in \mathrm{R}\}$.

Since the function is not reflected vertically or translated vertically, the graph of $y = 3\sqrt{-(x - 1)}$ extends up from $y = 0$, similar to the graph of $y = \sqrt{x}$. The range, $\{y \mid y \geq 0, y \in \mathrm{R}\}$, is unchanged by the transformations.

b) Express the function $y - 3 = -\sqrt{2x}$ in the form $y = a\sqrt{b(x - h)} + k$ to identify the value of each parameter.

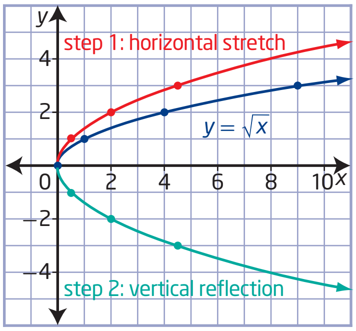
$$y - 3 = -\sqrt{2x}$$
$$y = -\sqrt{2x} + 3$$

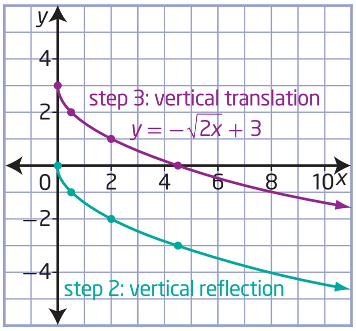
- $b = 2$ results in horizontal stretch by a factor of $\frac{1}{2}$ (step 1).
- $a = -1$ results in a reflection in the x-axis (step 2).
- $h = 0$, so the graph is not translated horizontally.
- $k = 3$ results in a vertical translation of 3 units up (step 3).

Apply these transformations either directly to the graph of $y = \sqrt{x}$ or to key points, and then sketch the transformed graph.

Method 1: Transform the Graph Directly

Use a sketch of $y = \sqrt{x}$ and apply the transformations to the curve one at a time.





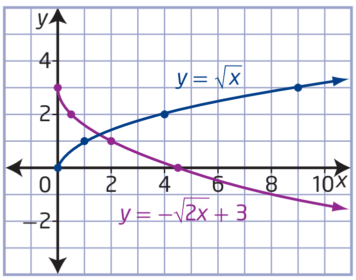
Method 2: Use Mapping Notation

Apply each transformation to the point (x, y) to determine a general mapping notation for the transformed function.

Transformation of $y = \sqrt{x}$	Mapping
Horizontal stretch by a factor of $\frac{1}{2}$	$(x, y) \rightarrow \left(\frac{1}{2}x, y\right)$
Reflection in the x-axis	$\left(\frac{1}{2}x, y\right) \rightarrow \left(\frac{1}{2}x, -y\right)$
Vertical translation of 3 units up	$\left(\frac{1}{2}x, -y\right) \rightarrow \left(\frac{1}{2}x, -y + 3\right)$

Choose key points on the graph of $y = \sqrt{x}$ and use the general mapping notation $(x, y) \rightarrow \left(\frac{1}{2}x, -y + 3\right)$ to determine their image points on the function $y - 3 = -\sqrt{2x}$.

$(0, 0) \rightarrow (0, 3)$
$(1, 1) \rightarrow (0.5, 2)$
$(4, 2) \rightarrow (2, 1)$
$(9, 3) \rightarrow (4.5, 0)$



Since there are no horizontal reflections or translations, the graph still extends to the right from $x = 0$. The domain, $\{x \mid x \geq 0, x \in \mathrm{R}\}$, is unchanged by the transformations as compared with $y = \sqrt{x}$.

The function is reflected vertically and then translated 3 units up, so the graph extends down from $y = 3$. The range is $\{y \mid y \leq 3, y \in \mathrm{R}\}$, which has changed as compared to $y = \sqrt{x}$.

Your Turn

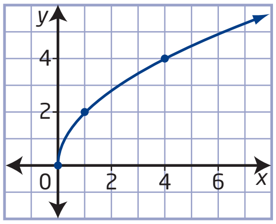
a) Sketch the graph of the function $y = -2\sqrt{x + 3} - 1$ by transforming the graph of $y = \sqrt{x}$.

b) Identify the domain and range of $y = \sqrt{x}$ and describe how they are affected by the transformations.

Example 3

Determine a Radical Function From a Graph

Mayleen is designing a symmetrical pattern. She sketches the curve shown and wants to determine its equation and the equation of its reflection in each quadrant. The graph is a transformation of the graph of $y = \sqrt{x}$. What are the equations of the four functions Mayleen needs to work with?



Solution

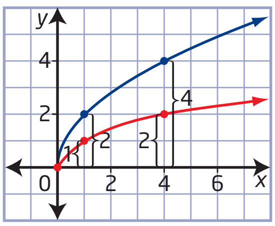
The base function $y = \sqrt{x}$ is not reflected or translated, but it is stretched. A radical function that involves a stretch can be obtained from either a vertical stretch or a horizontal stretch. Use an equation of the form $y = a\sqrt{x}$ or $y = \sqrt{bx}$ to represent the image function for each type of stretch.

Method 1: Compare Vertical or Horizontal Distances

Superimpose the graph of $y = \sqrt{x}$ and compare corresponding distances to determine the factor by which the function has been stretched.

View as a Vertical Stretch ($y = a\sqrt{x}$)

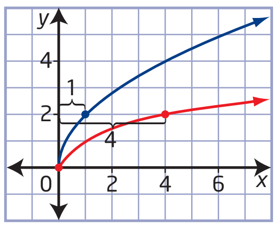
Each vertical distance is 2 times the corresponding distance for $y = \sqrt{x}$.



This represents a vertical stretch by a factor of 2, which means $a = 2$. The equation $y = 2\sqrt{x}$ represents the function.

View as a Horizontal Stretch ($y = \sqrt{bx}$)

Each horizontal distance is $\frac{1}{4}$ the corresponding distance for $y = \sqrt{x}$.



This represents a horizontal stretch by a factor of $\frac{1}{4}$, which means $b = 4$. The equation $y = \sqrt{4x}$ represents the function.

Express the equation of the function as either $y = 2\sqrt{x}$ or $y = \sqrt{4x}$.

Method 2: Substitute Coordinates of a Point
Use the coordinates of one point on the function, such as (1, 2), to determine the stretch factor.

View as a Vertical Stretch	**View as a Horizontal Stretch**
Substitute 1 for x and 2 for y in the equation $y = a\sqrt{x}$. Then, solve for a. $y = a\sqrt{x}$ $2 = a\sqrt{1}$ $2 = a(1)$ $2 = a$ The equation of the function is $y = 2\sqrt{x}$.	Substitute the coordinates (1, 2) in the equation $y = \sqrt{bx}$ and solve for b. $y = \sqrt{bx}$ $2 = \sqrt{b(1)}$ $2 = \sqrt{b}$ $2^2 = (\sqrt{b})^2$ $4 = b$ The equation can also be expressed as $y = \sqrt{4x}$.

Represent the function in simplest form by $y = 2\sqrt{x}$ or by $y = \sqrt{4x}$.

Determine the equations of the reflected curves using $y = 2\sqrt{x}$.
- A reflection in the y-axis results in the function $y = 2\sqrt{-x}$, since $b = -1$.
- A reflection in the x-axis results in $y = -2\sqrt{x}$, since $a = -1$.

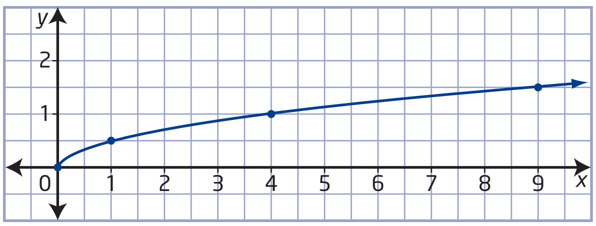
Reflecting these graphs into the third quadrant results in the function $y = -2\sqrt{-x}$.

Mayleen needs to use the equations $y = 2\sqrt{x}$, $y = 2\sqrt{-x}$, $y = -2\sqrt{x}$, and $y = -2\sqrt{-x}$. Similarly, she could use the equations $y = \sqrt{4x}$, $y = \sqrt{-4x}$, $y = -\sqrt{4x}$, and $y = -\sqrt{-4x}$.

Are the restrictions on the domain in each function consistent with the quadrant in which the curve lies?

Your Turn

a) Determine two forms of the equation for the function shown. The function is a transformation of the function $y = \sqrt{x}$.
b) Show algebraically that the two equations are equivalent.
c) What is the equation of the curve reflected in each quadrant?



Example 4

Model the Speed of Sound

Justin's physics textbook states that the speed, s, in metres per second, of sound in dry air is related to the air temperature, T, in degrees Celsius, by the function $s = 331.3\sqrt{1 + \frac{T}{273.15}}$.

a) Determine the domain and range in this context.

b) On the Internet, Justin finds another formula for the speed of sound, $s = 20\sqrt{T + 273}$. Use algebra to show that the two functions are approximately equivalent.

c) How is the graph of this function related to the graph of the base square root function? Which transformation do you predict will be the most noticeable on a graph?

d) Graph the function $s = 331.3\sqrt{1 + \frac{T}{273.15}}$ using technology.

e) Determine the speed of sound, to the nearest metre per second, at each of the following temperatures.

i) 20 °C (normal room temperature)

ii) 0 °C (freezing point of water)

iii) −63 °C (coldest temperature ever recorded in Canada)

iv) −89 °C (coldest temperature ever recorded on Earth)

Solution

a) Use the following inequality to determine the domain:

$$\begin{aligned} \text{radicand} &\geq 0 \\ 1 + \frac{T}{273.15} &\geq 0 \\ \frac{T}{273.15} &\geq -1 \\ T &\geq -273.15 \end{aligned}$$

The domain is $\{T \mid T \geq -273.15, T \in \mathrm{R}\}$. This means that the temperature must be greater than or equal to −273.15 °C, which is the lowest temperature possible and is referred to as absolute zero.

The range is $\{s \mid s \geq 0, s \in \mathrm{R}\}$, which means that the speed of sound is a non-negative value.

b) Rewrite the function from the textbook in simplest form.

$$s = 331.3\sqrt{1 + \frac{T}{273.15}}$$

$$s = 331.3\sqrt{\frac{273.15}{273.15} + \frac{T}{273.15}}$$

$$s = 331.3\sqrt{\frac{273.15 + T}{273.15}}$$

$$s = 331.3\frac{\sqrt{273.15 + T}}{\sqrt{273.15}}$$

$$s \approx 20\sqrt{T + 273}$$

How could you verify that these expressions are approximately equivalent?

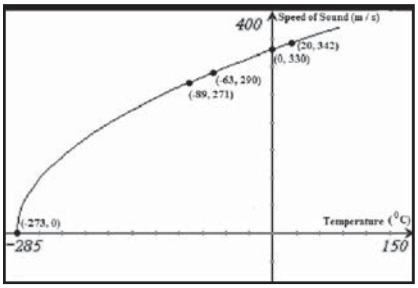
The function found on the Internet, $s = 20\sqrt{T + 273}$, is the approximate simplest form of the function in the textbook.

c) Analyse the transformations and determine the order in which they must be performed.

The graph of $s = \sqrt{T}$ is stretched vertically by a factor of about 20 and then translated about 273 units to the left. Translating 273 units to the left will be most noticeable on the graph of the function.

Are these transformations consistent with the domain and range?

d)



Are your answers to part c) confirmed by the graph?

e)

	Temperature (°C)	Approximate Speed of Sound (m/s)
i)	20	343
ii)	0	331
iii)	−63	291
iv)	−89	272

Your Turn

A company estimates its cost of production using the function $C(n) = 20\sqrt{n} + 1000$, where C represents the cost, in dollars, to produce n items.

a) Describe the transformations represented by this function as compared to $C = \sqrt{n}$.

b) Graph the function using technology. What does the shape of the graph imply about the situation?

c) Interpret the domain and range in this context.

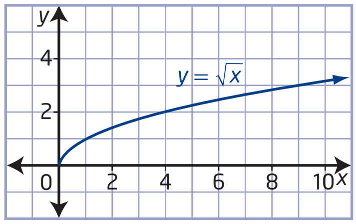
d) Use the graph to determine the expected cost to produce 12 000 items.

Did You Know?

Eureka, on Ellesmere Island, Nunavut, holds the North American record for the lowest-ever average monthly temperature, −47.9 °C in February 1979. For 18 days, the temperature stayed below −45 °C.

Key Ideas

- The base radical function is $y = \sqrt{x}$. Its graph has the following characteristics:
  - a left endpoint at (0, 0)
  - no right endpoint
  - the shape of half of a parabola
  - a domain of $\{x \mid x \geq 0, x \in \mathrm{R}\}$ and a range of $\{y \mid y \geq 0, y \in \mathrm{R}\}$



- You can graph radical functions of the form $y = a\sqrt{b(x - h)} + k$ by transforming the base function $y = \sqrt{x}$.

How does each parameter affect the graph of $y = \sqrt{x}$?

- You can analyse transformations to identify the domain and range of a radical function of the form $y = a\sqrt{b(x - h)} + k$.

Check Your Understanding

Practise

1. Graph each function using a table of values. Then, identify the domain and range.

a) $y = \sqrt{x - 1}$

b) $y = \sqrt{x + 6}$

c) $y = \sqrt{3 - x}$

d) $y = \sqrt{-2x - 5}$

2. Explain how to transform the graph of $y = \sqrt{x}$ to obtain the graph of each function. State the domain and range in each case.

a) $y = 7\sqrt{x - 9}$

b) $y = \sqrt{-x} + 8$

c) $y = -\sqrt{0.2x}$

d) $4 + y = \frac{1}{3}\sqrt{x + 6}$

3. Match each function with its graph.

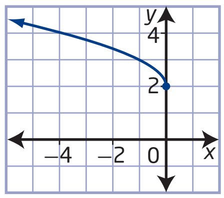
a) $y = \sqrt{x} - 2$

b) $y = \sqrt{-x} + 2$

c) $y = -\sqrt{x + 2}$

d) $y = -\sqrt{-(x - 2)}$

A



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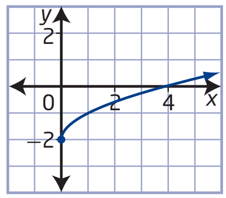

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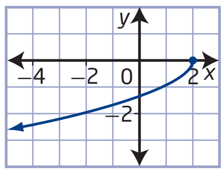

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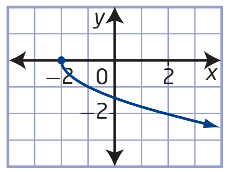


4. Write the equation of the radical function that results by applying each set of transformations to the graph of $y = \sqrt{x}$.

a) vertical stretch by a factor of 4, then horizontal translation of 6 units left

b) horizontal stretch by a factor of $\frac{1}{8}$, then vertical translation of 5 units down

c) horizontal reflection in the y-axis, then horizontal translation of 4 units right and vertical translation of 11 units up

d) vertical stretch by a factor of 0.25, vertical reflection in the x-axis, and horizontal stretch by a factor of 10

5. Sketch the graph of each function using transformations. State the domain and range of each function.

a) $f(x) = \sqrt{-x} - 3$

b) $r(x) = 3\sqrt{x + 1}$

c) $p(x) = -\sqrt{x - 2}$

d) $y - 1 = -\sqrt{-4(x - 2)}$

e) $m(x) = \sqrt{\frac{1}{2}x + 4}$

f) $y + 1 = \frac{1}{3}\sqrt{-(x + 2)}$

Apply

6. Consider the function $f(x) = \frac{1}{4}\sqrt{5x}$.

a) Identify the transformations represented by $f(x)$ as compared to $y = \sqrt{x}$.

b) Write two functions equivalent to $f(x)$: one of the form $y = a\sqrt{x}$ and the other of the form $y = \sqrt{bx}$

c) Identify the transformation(s) represented by each function you wrote in part b).

d) Use transformations to graph all three functions. How do the graphs compare?

7. a) Express the radius of a circle as a function of its area.

b) Create a table of values and a graph to illustrate the relationship that this radical function represents.

8. For an observer at a height of h feet above the surface of Earth, the approximate distance, d, in miles, to the horizon can be modelled using the radical function $d = \sqrt{1.50h}$.

a) Use the language of transformations to describe how to obtain the graph from the base square root graph.

b) Determine an approximate equivalent function of the form $d = a\sqrt{h}$ for the function. Which form of the function do you prefer, and why?

c) A lifeguard on a tower is looking out over the water with binoculars. How far can she see if her eyes are 20 ft above the level of the water? Express your answer to the nearest tenth of a mile.

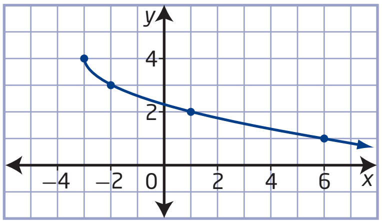
9. The function $4 - y = \sqrt{3x}$ is translated 9 units up and reflected in the x-axis.

a) Without graphing, determine the domain and range of the image function.

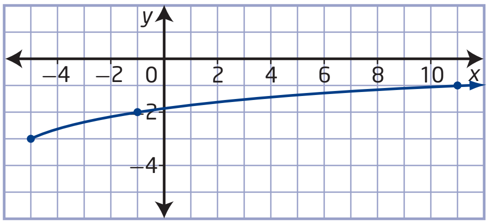
b) Compared to the base function, $y = \sqrt{x}$, by how many units and in which direction has the given function been translated horizontally? vertically?

10. For each graph, write the equation of a radical function of the form $y = a\sqrt{b(x - h)} + k$.

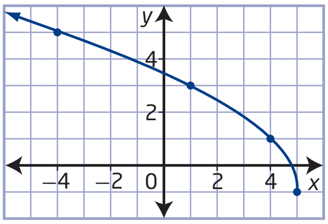
a)



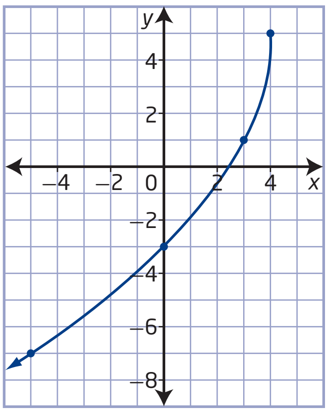
b)



c)



d)



11. Write the equation of a radical function with each domain and range.

a) $\{x \mid x \geq 6, x \in R\}, \{y \mid y \geq 1, y \in R\}$

b) $\{x \mid x \geq -7, x \in R\}, \{y \mid y \leq -9, y \in R\}$

c) $\{x \mid x \leq 4, x \in R\}, \{y \mid y \geq -3, y \in R\}$

d) $\{x \mid x \leq -5, x \in R\}, \{y \mid y \leq 8, y \in R\}$

12. Agronomists use radical functions to model and optimize corn production. One factor they analyse is how the amount of nitrogen fertilizer applied affects the crop yield. Suppose the function $Y(n) = 760\sqrt{n} + 2000$ is used to predict the yield, Y, in kilograms per hectare, of corn as a function of the amount, n, in kilograms per hectare, of nitrogen applied to the crop.

a) Use the language of transformations to compare the graph of this function to the graph of $y = \sqrt{n}$.

b) Graph the function using transformations.

c) Identify the domain and range.

d) What do the shape of the graph, the domain, and the range tell you about this situation? Are the domain and range realistic in this context? Explain.

Did You Know?

Over 6300 years ago, the Indigenous people in the area of what is now Mexico domesticated and cultivated several varieties of corn. The cultivation of corn is now global.

13. A manufacturer wants to predict the consumer interest in a new smart phone. The company uses the function $P(d) = -2\sqrt{-d} + 20$ to model the number, P, in millions, of pre-orders for the phone as a function of the number, d, of days before the phone's release date.

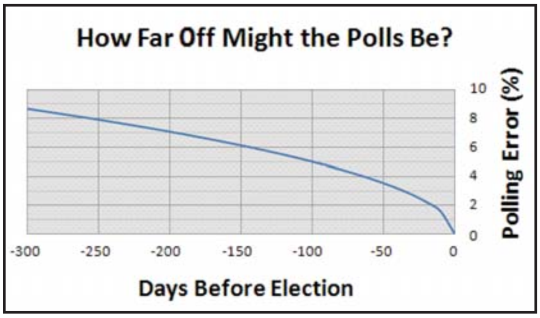
a) What are the domain and range and what do they mean in this situation?

b) Identify the transformations represented by the function as compared to $y = \sqrt{d}$.

c) Graph the function and explain what the shape of the graph indicates about the situation.

d) Determine the number of pre-orders the manufacturer can expect to have 30 days before the release date.

14. During election campaigns, campaign managers use surveys and polls to make projections about the election results. One campaign manager uses a radical function to model the possible error in polling predictions as a function of the number of days until the election, as shown in the graph.

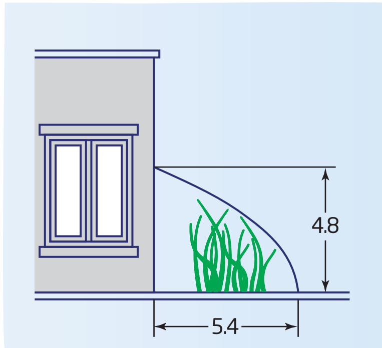


a) Explain what the graph shows about the accuracy of polls before elections.

b) Determine an equation to represent the function. Show how you developed your answer.

c) Describe the transformations that the function represents as compared to $y = \sqrt{x}$.

15. While meeting with a client, a manufacturer of custom greenhouses sketches a greenhouse in the shape of the graph of a radical function. What equation could the manufacturer use to represent the shape of the greenhouse roof?



Did You Know?

People living in the Arctic are starting to use greenhouses to grow some of their food. There are greenhouse societies in both Iqaluit, Nunavut and Inuvik, Northwest Territories that grow beans, lettuce, carrots, tomatoes, and herbs.

Web Link

To learn more about greenhouse communities in the Arctic, go to www.mcgrawhill.ca/school/learningcentres and follow the links.

16. Determine the equation of a radical function with

a) endpoint at (2, 5) and passing through the point (6, 1)

b) endpoint at (3, −2) and an x-intercept with a value of −6

17. The Penrose method is a system for giving voting powers to members of assemblies or legislatures based on the square root of the number of people that each member represents, divided by 1000. Consider a parliament that represents the people of the world and how voting power might be given to different nations. The table shows the estimated populations of Canada and the three most populous and the three least populous countries in the world.

Country	Population
China	1 361 513 000
India	1 251 696 000
United States	325 540 000
Canada	35 100 000
Tuvalu	11 000
Nauru	10 000
Vatican City	1 000

a) Share your answers to the following two questions with a classmate and explain your thinking:

- Which countries might feel that a "one nation, one vote" system is an unfair way to allocate voting power?
- Which countries might feel that a "one person, one vote" system is unfair?

b) What percent of the voting power would each nation listed above have under a "one person, one vote" system, assuming a world population of approximately 7.302 billion?

c) If x represents the population of a country and $V(x)$ represents its voting power, what function could be written to represent the Penrose method?

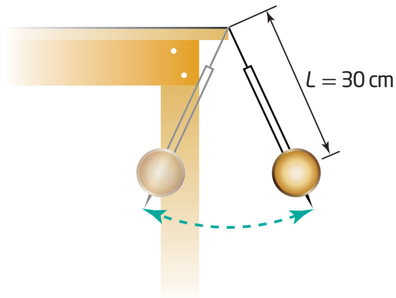
d) Under the Penrose method, the sum of the world voting power using the given data is approximately 765. What percent of the voting power would this system give each nation in the table?

e) Why might the Penrose method be viewed as a compromise for allocating voting power?

18. MINI LAB The period of a pendulum is the time for one complete swing back and forth. As long as the initial swing angle is kept relatively small, the period of a pendulum is related to its length by a radical function.

Materials

- thread
- washer or other suitable mass
- tape
- ruler
- stopwatch or timer



Step 1 Tie a length of thread to a washer or other mass. Tape the thread to the edge of a table or desk top so that the length between the pivot point and the centre of the washer is 30 cm.

Step 2 Pull the mass to one side and allow it to swing freely. Measure the total time for 10 complete swings back and forth and then divide by 10 to determine the period for this length. Record the length and period in a table.

Step 3 Repeat steps 1 and 2 using lengths of 25 cm, 20 cm, 15 cm, 10 cm, 5 cm, and 3 cm (and shorter distances if possible).

Step 4 Create a scatter plot showing period as a function of length. Draw a smooth curve through or near the points. Does it appear to be a radical function? Justify your answer.

Step 5 What approximate transformation(s) to the graph of $y = \sqrt{x}$ would produce your result? Write a radical function that approximates the graph, where T represents the period and L represents the length of the pendulum.

Extend

19. The inverse of $f(x) = \sqrt{x}$ is $f^{-1}(x) = x^2, x \geq 0$.

a) Graph both functions, and use them to explain why the restriction is necessary on the domain of the inverse function.

b) Determine the equation, including any restrictions, of the inverse of each of the following functions.

i) $g(x) = -\sqrt{x - 5}$

ii) $h(x) = \sqrt{-x} + 3$

iii) $j(x) = \sqrt{2x - 7} - 6$

20. If $f(x) = \frac{5}{8}\sqrt{-\frac{7}{12}x}$ and $g(x) = -\frac{2}{5}\sqrt{6(x + 3)} - 4$, what transformations could you apply to the graph of $f(x)$ to create the graph of $g(x)$?

Create Connections

C1 Which parameters in $y = a\sqrt{b(x - h)} + k$ affect the domain of $y = \sqrt{x}$? Which parameters affect the range? Explain, using examples.

C2 Sarah claims that any given radical function can be simplified so that there is no value of b, only a value of a. Is she correct? Explain, using examples.

C3 Compare and contrast the process of graphing a radical function using transformations with graphing a quadratic function using transformations.

C4 **MINI LAB** The Wheel of Theodorus, or Square Root Spiral, is a geometric construction that contains line segments with length equal to the square root of any whole number.

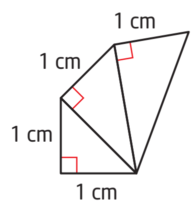
Materials

- ruler, drafting square, or other object with a right angle
- millimetre ruler

Step 1 Create an isosceles right triangle with legs that are each 1 cm long. Mark one end of the hypotenuse as point C. What is the length of the hypotenuse, expressed as a radical?

Step 2 Use the hypotenuse of the first triangle as one leg of a new right triangle. Draw a length of 1 cm as the other leg, opposite point C. What is the length of the hypotenuse of this second triangle, expressed as a radical?

Step 3 Continue to create right triangles, each time using the hypotenuse of the previous triangle as a leg of the next triangle, and a length of 1 cm as the other leg (drawn so that the 1-cm leg is opposite point C). Continue the spiral until you would overlap the initial base.



Step 4 Create a table to represent the length of the hypotenuse as a function of the triangle number (first, second, third triangle in the pattern, etc.). Express lengths both in exact radical form and in approximate decimal form.

Step 5 Write an equation to represent this function, where L represents the hypotenuse length and n represents the triangle number. Does the equation involve any transformations on the base square root function? Explain.

2.2

Square Root of a Function

Focus on...

- sketching the graph of $y = \sqrt{f(x)}$ given the graph of $y = f(x)$
- explaining strategies for graphing $y = \sqrt{f(x)}$ given the graph of $y = f(x)$
- comparing the domains and ranges of the functions $y = f(x)$ and $y = \sqrt{f(x)}$, and explaining any differences

The Pythagorean theorem is often applied by engineers. They use right triangles in the design of large domes, bridges, and other structures because the triangle is a strong support unit. For example, a truss bridge consists of triangular units of steel beams connected together to support the bridge deck.

You are already familiar with the square root operation (and its effect on given values) in the Pythagorean theorem. How does the square root operation affect the graph of the function? If you are given the graph of a function, what does the graph of the square root of that function look like?

Web Link

For more information about how triangles are fundamental to the design of domes, go to www.mcgrawhill.ca/school/learningcentres and follow the links.

Truss bridge over the Bow River in Morley, Alberta located on Chiniki First Nation territory.

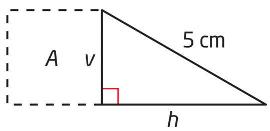
Investigate Related Functions: $y = f(x)$ and $y = \sqrt{f(x)}$

Materials

- grid paper and ruler, or graphing calculator
- dynamic geometry or graphing software (optional)

A: Right Triangles, Area, and Length

1. Draw several right triangles with a hypotenuse of 5 cm and legs of various lengths. For each triangle, label the legs as v and h.



2. a) Write an equation for the length of v as a function of h. Graph the function using an appropriate domain for the situation.

b) Compare the measured values for side v in the triangles you drew to the calculated values of v from your graph.

3. a) Draw a square on side v of each triangle. Let the area of this square be A, and write an equation for A as a function of h.

b) Graph the area function.

Reflect and Respond

4. a) How are the equations of the two functions related?

b) How do the domains of the two functions compare?

c) What is the relationship between the ranges of the two functions?

B: Compare a Function and Its Square Root

5. Consider the functions $y = 2x + 4$ and $y = \sqrt{2x + 4}$.

a) Describe the relationship between the equations for these two functions.

b) Graph the two functions, and note any connections between the two graphs.

c) Compare the values of y for the same values of x. How are they related?

6. a) Create at least two more pairs of functions that share the same relationship as those in step 5.

b) Compare the tables and graphs of each pair of functions.

Reflect and Respond

7. Consider pairs of functions where one function is the square root of the other function.

a) How do the domains compare? Explain why you think there are differences.

b) How are the values of y related for pairs of functions like these?

c) What differences occur in the ranges, and why do you think they occur?

8. How might you use the connections you have identified in this investigation as a method of graphing $y = \sqrt{f(x)}$ if you are given the graph of $y = f(x)$?

Link the Ideas

You can determine how two functions, $y = f(x)$ and $y = \sqrt{f(x)}$, are related by comparing how the values of y are calculated:

- For $y = 2x + 1$, multiply x by 2 and add 1.
- For $y = \sqrt{2x + 1}$, multiply x by 2, add 1, and take the square root.

The two functions start with the same two operations, but the function $y = \sqrt{2x + 1}$ has the additional step of taking the square root. For any value of x, the resulting value of y for $y = \sqrt{2x + 1}$ is the square root of the value of y for $y = 2x + 1$, as shown in the table.

x	$y = 2x + 1$	$y = \sqrt{2x + 1}$
0	1	1
4	9	3
12	25	5
24	49	7
⋮	⋮	⋮

The function $y = \sqrt{2x + 1}$ represents the **square root of the function** $y = 2x + 1$.

square root of a function

- the function $y = \sqrt{f(x)}$ is the square root of the function $y = f(x)$
- $y = \sqrt{f(x)}$ is only defined for $f(x) \geq 0$

Example 1

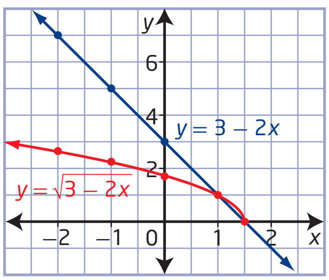
Compare Graphs of a Linear Function and the Square Root of the Function

a) Given $f(x) = 3 - 2x$, graph the functions $y = f(x)$ and $y = \sqrt{f(x)}$.
b) Compare the two functions.

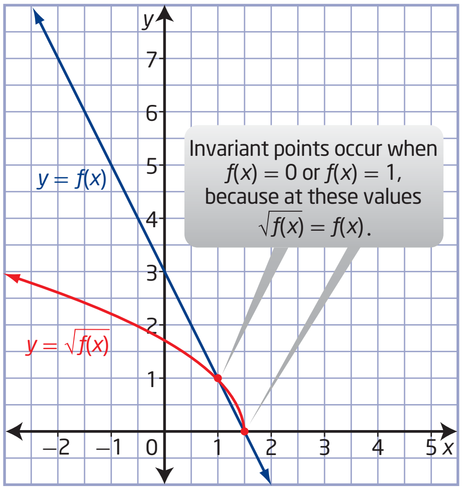
Solution

a) Use a table of values to graph $y = 3 - 2x$ and $y = \sqrt{3 - 2x}$.

x	$y = 3 - 2x$	$y = \sqrt{3 - 2x}$
–2	7	$\sqrt{7}$
–1	5	$\sqrt{5}$
0	3	$\sqrt{3}$
1	1	1
1.5	0	0



b) Compare the graphs.



Why is the graph of $y = \sqrt{f(x)}$ above the graph of $y = f(x)$ for values of y between 0 and 1? Will this always be true?

For $y = f(x)$, the domain is $\{x \mid x \in \mathrm{R}\}$ and the range is $\{y \mid y \in \mathrm{R}\}$.
For $y = \sqrt{f(x)}$, the domain is $\{x \mid x \leq 1.5, x \in \mathrm{R}\}$ and the range is $\{y \mid y \geq 0, y \in \mathrm{R}\}$.

Invariant points occur at (1, 1) and (1.5, 0).

How does the domain of the graph of $y = \sqrt{f(x)}$ relate to the restrictions on the variable in the radicand? How could you determine the domain algebraically?

Your Turn

a) Given $g(x) = 3x + 6$, graph the functions $y = g(x)$ and $y = \sqrt{g(x)}$.
b) Identify the domain and range of each function and any invariant points.

Relative Locations of $y = f(x)$ and $y = \sqrt{f(x)}$

The domain of $y = \sqrt{f(x)}$ consists only of the values in the domain of $f(x)$ for which $f(x) \geq 0$.

The range of $y = \sqrt{f(x)}$ consists of the square roots of the values in the range of $y = f(x)$ for which $\sqrt{f(x)}$ is defined.

The graph of $y = \sqrt{f(x)}$ exists only where $f(x) \geq 0$. You can predict the location of $y = \sqrt{f(x)}$ relative to $y = f(x)$ using the values of $f(x)$.

Value of $f(x)$	$f(x) < 0$	$f(x) = 0$	$0 < f(x) < 1$	$f(x) = 1$	$f(x) > 1$
Relative Location of Graph of $y = \sqrt{f(x)}$	The graph of $y = \sqrt{f(x)}$ is undefined.	The graphs of $y = \sqrt{f(x)}$ and $y = f(x)$ intersect on the x-axis.	The graph of $y = \sqrt{f(x)}$ is above the graph of $y = f(x)$.	The graph of $y = \sqrt{f(x)}$ intersects the graph of $y = f(x)$.	The graph of $y = \sqrt{f(x)}$ is below the graph of $y = f(x)$.

Example 2

Compare the Domains and Ranges of $y = f(x)$ and $y = \sqrt{f(x)}$

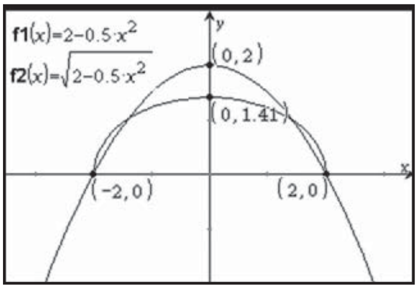
Identify and compare the domains and ranges of the functions in each pair.

a) $y = 2 - 0.5x^2$ and $y = \sqrt{2 - 0.5x^2}$
b) $y = x^2 + 5$ and $y = \sqrt{x^2 + 5}$

Solution

a) Method 1: Analyse Graphically
Since the function $y = 2 - 0.5x^2$ is a quadratic function, its square root, $y = \sqrt{2 - 0.5x^2}$, cannot be expressed in the form $y = a\sqrt{b(x - h)} + k$. It cannot be graphed by transforming $y = \sqrt{x}$.

Both graphs can be created using technology. Use the *maximum* and *minimum* or equivalent features to find the coordinates of points necessary to determine the domain and range.



The graph of $y = 2 - 0.5x^2$ extends from (0, 2) down and to the left and right infinitely. Its domain is $\{x \mid x \in \mathrm{R}\}$, and its range is $\{y \mid y \leq 2, y \in \mathrm{R}\}$.

The graph of $y = \sqrt{2 - 0.5x^2}$ includes values of x from -2 to 2 inclusive, so its domain is $\{x \mid -2 \leq x \leq 2, x \in \mathrm{R}\}$. The graph covers values of y from 0 to approximately 1.41 inclusive, so its approximate range is $\{y \mid 0 \leq y \leq 1.41, y \in \mathrm{R}\}$.

To determine the exact value that 1.41 represents, you need to analyse the function algebraically.

The domain and range of $y = \sqrt{2 - 0.5x^2}$ are subsets of the domain and range of $y = 2 - 0.5x^2$.

Method 2: Analyse Key Points
Use the locations of any intercepts and the maximum value or minimum value to determine the domain and range of each function.

Function	$y = 2 - 0.5x^2$	$y = \sqrt{2 - 0.5x^2}$
x-Intercepts	-2 and 2	-2 and 2
y-Intercept	2	$\sqrt{2}$
Maximum Value	2	$\sqrt{2}$
Minimum Value	none	0

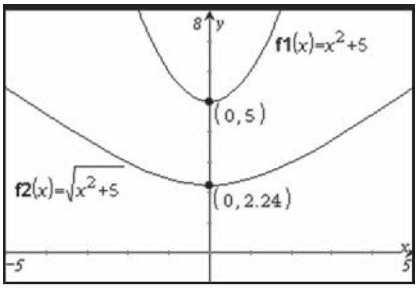
How can you justify this information algebraically?

Quadratic functions are defined for all real numbers. So, the domain of $y = 2 - 0.5x^2$ is $\{x \mid x \in R\}$. Since the maximum value is 2, the range of $y = 2 - 0.5x^2$ is $\{y \mid y \leq 2, y \in R\}$.

The locations of the x-intercepts of $y = \sqrt{2 - 0.5x^2}$ mean that the function is defined for $-2 \leq x \leq 2$. So, the domain is $\{x \mid -2 \leq x \leq 2, x \in R\}$. Since $y = \sqrt{2 - 0.5x^2}$ has a minimum value of 0 and a maximum value of $\sqrt{2}$, the range is $\{y \mid 0 \leq y \leq \sqrt{2}, y \in R\}$.

b) Method 1: Analyse Graphically

Graph the functions $y = x^2 + 5$ and $y = \sqrt{x^2 + 5}$ using technology.



Both functions extend infinitely to the left and the right, so the domain of each function is $\{x \mid x \in R\}$.

The range of $y = x^2 + 5$ is $\{y \mid y \geq 5, y \in R\}$.

The range of $y = \sqrt{x^2 + 5}$ is approximately $\{y \mid y \geq 2.24, y \in R\}$.

Method 2: Analyse Key Points

Use the locations of any intercepts and the maximum value or minimum value to determine the domain and range of each function.

Function	$y = x^2 + 5$	$y = \sqrt{x^2 + 5}$
x-Intercepts	none	none
y-Intercept	5	$\sqrt{5}$
Maximum Value	none	none
Minimum Value	5	$\sqrt{5}$

Quadratic functions are defined for all real numbers. So, the domain of $y = x^2 + 5$ is $\{x \mid x \in R\}$. Since the minimum value is 5, the range of $y = x^2 + 5$ is $\{y \mid y \geq 5, y \in R\}$.

Since $y = \sqrt{x^2 + 5}$ has no x-intercepts, the function is defined for all real numbers. So, the domain is $\{x \mid x \in R\}$. Since $y = \sqrt{x^2 + 5}$ has a minimum value of $\sqrt{5}$ and no maximum value, the range is $\{y \mid y \geq \sqrt{5}, y \in R\}$.

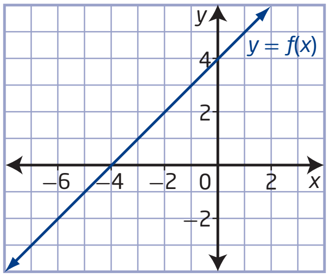
Your Turn

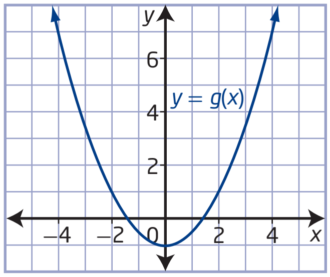
Identify and compare the domains and ranges of the functions $y = x^2 - 1$ and $y = \sqrt{x^2 - 1}$. Verify your answers.

Example 3

Graph the Square Root of a Function From the Graph of the Function

Using the graphs of $y = f(x)$ and $y = g(x)$, sketch the graphs of $y = \sqrt{f(x)}$ and $y = \sqrt{g(x)}$.





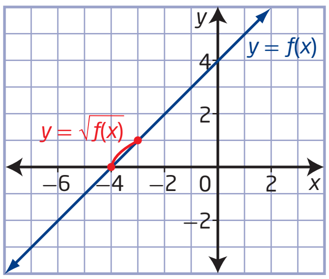
Solution

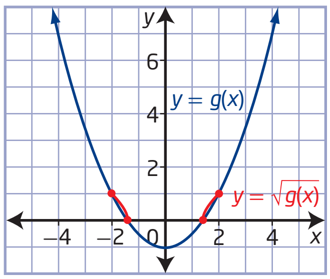
Sketch each graph by locating key points, including invariant points, and determining the image points on the graph of the square root of the function.

Step 1: Locate invariant points on $y = f(x)$ and $y = g(x)$. When graphing the square root of a function, invariant points occur at $y = 0$ and $y = 1$.

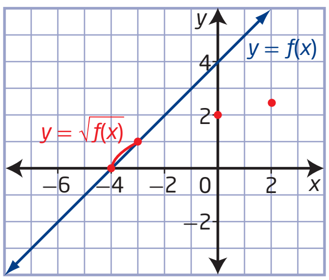
What is significant about $y = 0$ and $y = 1$? Does this apply to all graphs of functions and their square roots? Why?

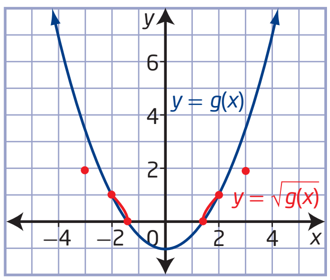
Step 2: Draw the portion of each graph between the invariant points for values of $y = f(x)$ and $y = g(x)$ that are positive but less than 1. Sketch a smooth curve *above* those of $y = f(x)$ and $y = g(x)$ in these intervals.





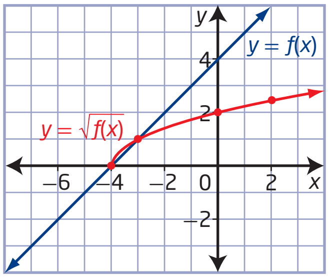
Step 3: Locate other key points on $y = f(x)$ and $y = g(x)$ where the values are greater than 1. Transform these points to locate image points on the graphs of $y = \sqrt{f(x)}$ and $y = \sqrt{g(x)}$.

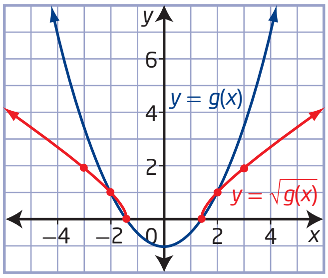




How can a value of *y* be mapped to a point on the square root of the function?

Step 4: Sketch smooth curves between the image points; they will be below those of $y = f(x)$ and $y = g(x)$ in the remaining intervals. Recall that graphs of $y = \sqrt{f(x)}$ and $y = \sqrt{g(x)}$ do not exist in intervals where $y = f(x)$ and $y = g(x)$ are negative (below the x-axis).

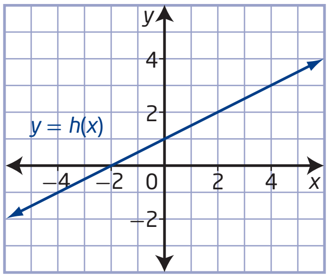




Where is the square root of a function above the original function? Where is it below? Where are they equal? Where are the endpoints on a graph of the square root of a function? Why?

Your Turn

Using the graph of $y = h(x)$, sketch the graph of $y = \sqrt{h(x)}$.



Key Ideas

- You can use values of $f(x)$ to predict values of $\sqrt{f(x)}$ and to sketch the graph of $y = \sqrt{f(x)}$.
- The key values to consider are $f(x) = 0$ and $f(x) = 1$.
- The domain of $y = \sqrt{f(x)}$ consists of all values in the domain of $f(x)$ for which $f(x) \geq 0$.
- The range of $y = \sqrt{f(x)}$ consists of the square roots of all values in the range of $f(x)$ for which $f(x)$ is defined.
- The y-coordinates of the points on the graph of $y = \sqrt{f(x)}$ are the square roots of the y-coordinates of the corresponding points on the original function $y = f(x)$.

What do you know about the graph of $y = \sqrt{f(x)}$ at $f(x) = 0$ and $f(x) = 1$? How do the graphs of $y = f(x)$ and $y = \sqrt{f(x)}$ compare on either side of these locations?

Check Your Understanding

Practise

1. Copy and complete the table.

$f(x)$	$\sqrt{f(x)}$
36	
	0.03
1	
−9	
	1.6
0	

2. For each point on the graph of $y = f(x)$, does a corresponding point on the graph of $y = \sqrt{f(x)}$ exist? If so, state the coordinates (rounded to two decimal places, if necessary).

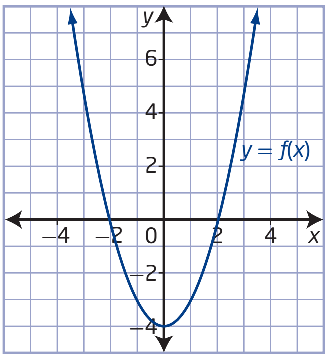
a) $(4, 12)$ **b)** $(-2, 0.4)$

c) $(10, -2)$ **d)** $(0.09, 1)$

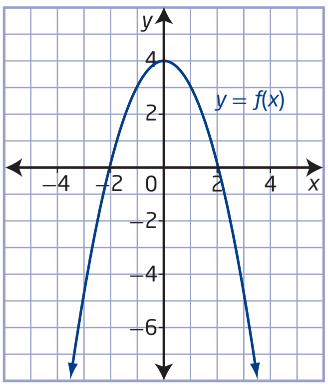
e) $(-5, 0)$ **f)** (m, n)

3. Match each graph of $y = f(x)$ to the corresponding graph of $y = \sqrt{f(x)}$.

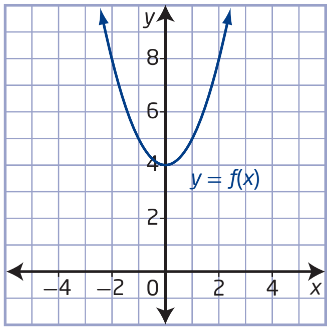
a)



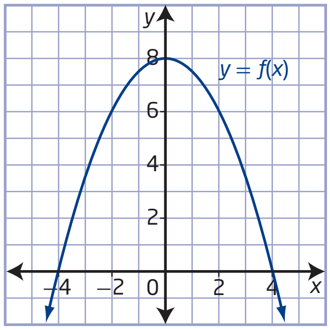
b)



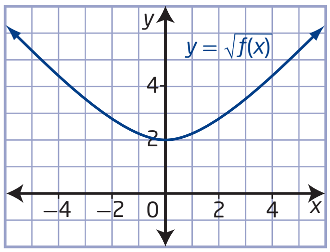
c)



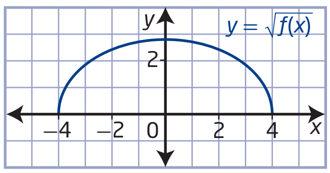
d)



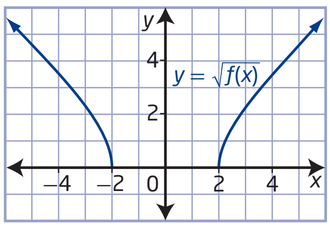
A



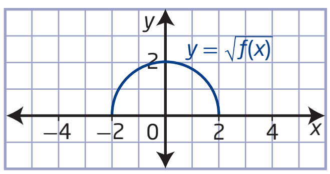
B



C



D



4. a) Given $f(x) = 4 - x$, graph the functions $y = f(x)$ and $y = \sqrt{f(x)}$.
   b) Compare the two functions and explain how their values are related.
   c) Identify the domain and range of each function, and explain any differences.

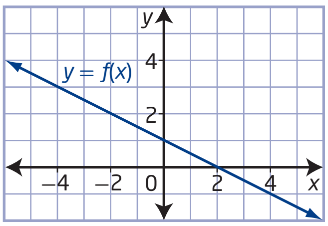
5. Determine the domains and ranges of the functions in each pair graphically and algebraically. Explain why the domains and ranges differ.
   a) $y = x - 2$, $y = \sqrt{x - 2}$
   b) $y = 2x + 6$, $y = \sqrt{2x + 6}$
   c) $y = -x + 9$, $y = \sqrt{-x + 9}$
   d) $y = -0.1x - 5$, $y = \sqrt{-0.1x - 5}$

6. Identify and compare the domains and ranges of the functions in each pair.
   a) $y = x^2 - 9$ and $y = \sqrt{x^2 - 9}$
   b) $y = 2 - x^2$ and $y = \sqrt{2 - x^2}$
   c) $y = x^2 + 6$ and $y = \sqrt{x^2 + 6}$
   d) $y = 0.5x^2 + 3$ and $y = \sqrt{0.5x^2 + 3}$

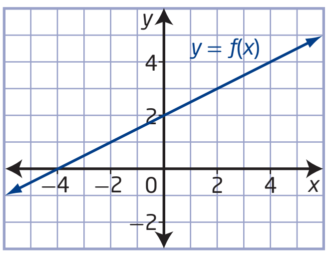
7. For each function, identify and explain any differences in the domains and ranges of $y = f(x)$ and $y = \sqrt{f(x)}$.
   a) $f(x) = x^2 - 25$
   b) $f(x) = x^2 + 3$
   c) $f(x) = 32 - 2x^2$
   d) $f(x) = 5x^2 + 50$

8. Using each graph of $y = f(x)$, sketch the graph of $y = \sqrt{f(x)}$.

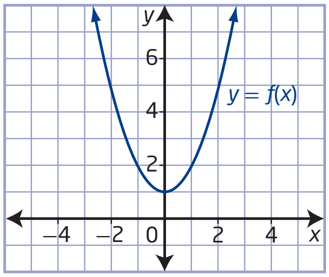
a)



b)



c)

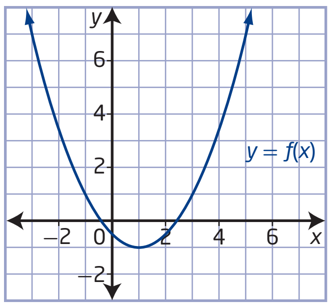


Apply

9. a) Use technology to graph each function and identify the domain and range.
      i) $f(x) = x^2 + 4$
      ii) $g(x) = x^2 - 4$
      iii) $h(x) = -x^2 + 4$
      iv) $j(x) = -x^2 - 4$
   b) Graph the square root of each function in part a) using technology.
   c) What do you notice about the graph of $y = \sqrt{j(x)}$? Explain this observation based on the graph of $y = j(x)$. Then, explain this observation algebraically.
   d) In general, how are the domains of the functions in part a) related to the domains of the functions in part b)? How are the ranges related?

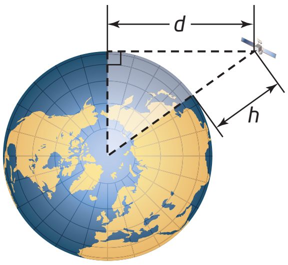
10. a) Identify the domains and ranges of $y = x^2 - 4$ and $y = \sqrt{x^2 - 4}$.
    b) Why is $y = \sqrt{x^2 - 4}$ undefined over an interval? How does this affect the domain of the function?

11. The graph of $y = f(x)$ is shown.



   a) Sketch the graph of $y = \sqrt{f(x)}$, and explain the strategy you used.
   b) State the domain and range of each function, and explain how the domains and the ranges are related.

12. For relatively small heights above Earth, a simple radical function can be used to approximate the distance to the horizon.



a) If Earth's radius is assumed to be 6378 km, determine the equation for the distance, d, in kilometres, to the horizon for an object that is at a height of h kilometres above Earth's surface.

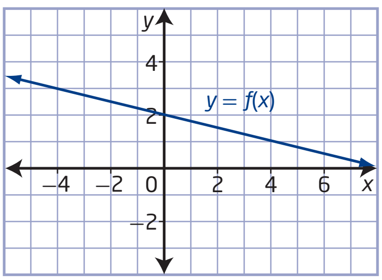
b) Identify the domain and range of the function.

c) How can you use a graph of the function to find the distance to the horizon for a satellite that is 800 km above Earth's surface?

d) If the function from part a) were just an arbitrary mathematical function rather than in this context, would the domain or range be any different? Explain.

13. a) When determining whether the graph shown represents a function or the square root of the function, Chris states, "it must be the function $y = f(x)$ because the domain consists of negative values, and the square root of a function $y = \sqrt{f(x)}$ is not defined for negative values."

Do you agree with Chris's answer? Why?



b) Describe how you would determine whether a graph shows the function or the square root of the function.

14. The main portion of an iglu (Inuit spelling of the English word igloo) is approximately hemispherical in shape.

a) For an iglu with diameter 3.6 m, determine a function that gives the vertical height, v, in metres, in terms of the horizontal distance, h, in metres, from the centre.

b) What are the domain and range of this function, and how are they related to the situation?

c) What is the height of this iglu at a point 1 m in from the bottom edge of the wall?

Did You Know?

An iglu is actually built in a spiral from blocks cut from inside the iglu floor space. Half the floor space is left as a bed platform in large iglus. This traps cold air below the sleeping area.

15. **MINI LAB** Investigate how the constants in radical functions affect their graphs, domains, and ranges.

Step 1 Graph the function $y = \sqrt{a^2 - x^2}$ for various values of a. If you use graphing software, you may be able to create sliders that allow you to vary the value of a and dynamically see the resulting changes in the graph.

Step 2 Describe how the value of a affects the graph of the function and its domain and range.

Step 3 Choose one value of a and write an equation for the reflection of this function in the x-axis. Graph both functions and describe the graph.

Step 4 Repeat steps 1 to 3 for the function $y = \sqrt{a^2 + x^2}$ as well as another square root of a function involving x^2.

Extend

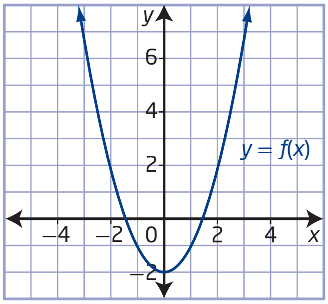
16. If (−24, 12) is a point on the graph of the function $y = f(x)$, identify one point on the graph of each of the following functions.

a) $y = \sqrt{4f(x + 3)}$

b) $y = -\sqrt{f(4x)} + 12$

c) $y = -2\sqrt{f(-(x - 2)) - 4} + 6$

17. Given the graph of the function $y = f(x)$, sketch the graph of each function.



a) $y = 2\sqrt{f(x)} - 3$

b) $y = -\sqrt{2f(x - 3)}$

c) $y = \sqrt{-f(2x) + 3}$

d) $y = \sqrt{2f(-x) - 3}$

18. Explain your strategy for completing #17b).

19. Develop a formula for radius as a function of surface area for

a) a cylinder with equal diameter and height

b) a cone with height three times its diameter

Create Connections

C1 Write a summary of your strategy for graphing the function $y = \sqrt{f(x)}$ if you are given only the graph of $y = f(x)$.

C2 Explain how the relationship between the two equations $y = 16 - 4x$ and $y = \sqrt{16 - 4x}$ is connected to the relationship between their graphs.

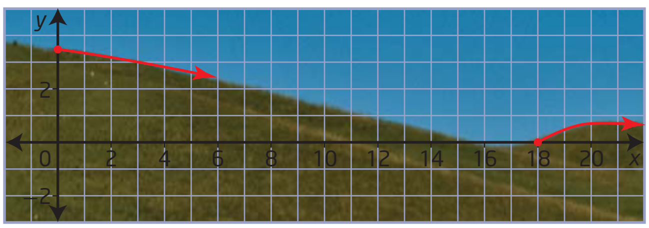
C3 Is it possible to completely graph the function $y = f(x)$ given only the graph of $y = \sqrt{f(x)}$? Discuss this with a classmate and share several examples that you create. Write a summary of your conclusions.

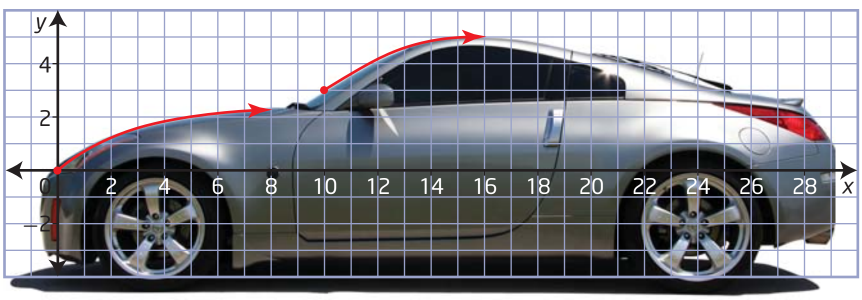
C4 **a)** Given $f(x) = (x - 1)^2 - 4$, graph the functions $y = f(x)$ and $y = \sqrt{f(x)}$.

b) Compare the two functions and explain how their values are related using several points on each graph.

Project Corner — Form Follows Function

- What radical functions are represented by the curves drawn on each image?





2.3

Solving Radical Equations Graphically

Focus on…

- relating the roots of radical equations and the *x*-intercepts of the graphs of radical functions
- determining approximate solutions of radical equations graphically

Parachutes slow the speed of falling objects by greatly increasing the drag force of the air. Manufacturers must make careful calculations to ensure that their parachutes are large enough to create enough drag force to allow parachutists to descend at a safe speed, but not so large that they are impractical. Radical equations can be used to relate the area of a parachute to the descent speed and mass of the object it carries, allowing parachute designers to ensure that their designs are reliable.

Did You Know?

French inventor Louis Sébastien-Lenormand introduced the first practical parachute in 1783.

Investigate Solving Radical Equations Graphically

Materials

- graphing calculator or graphing software

The radical equation $\sqrt{x - 4} = 5$ can be solved in several ways.

1. a) Discuss with a classmate how you might solve the equation graphically. Could you use more than one graphical method?

b) Write step-by-step instructions that explain how to use your method(s) to determine the solution to the radical equation.

c) Use your graphical method(s) to solve the equation.

2. a) Describe one method of solving the equation algebraically.

b) Use this method to determine the solution.

c) How might you verify your solution algebraically?

d) Share your method and solution with those of another pair and discuss any similarities and differences.

Reflect and Respond

3. a) How does the solution you found graphically compare with the one you found algebraically?

b) Will a graphical solution always match an algebraic solution? Discuss your answer with a classmate and explain your thoughts.

4. Do you prefer an algebraic or a graphical method for solving a radical equation like this one? Explain why.

Link the Ideas

You can solve many types of equations algebraically and graphically. Algebraic solutions sometimes produce extraneous roots, whereas graphical solutions do not produce extraneous roots. However, algebraic solutions are generally exact while graphical solutions are often approximate. You can solve equations, including radical equations, graphically by identifying the x-intercepts of the graph of the corresponding function.

Example 1

Relate Roots and x-Intercepts

a) Determine the root(s) of $\sqrt{x + 5} - 3 = 0$ algebraically.

b) Using a graph, determine the x-intercept(s) of the graph of $y = \sqrt{x + 5} - 3$.

c) Describe the connection between the root(s) of the equation and the x-intercept(s) of the graph of the function.

Solution

a) Identify any restrictions on the variable in the radical.

$$x + 5 \geq 0$$
$$x \geq -5$$

To solve a radical equation algebraically, first isolate the radical.

$$\sqrt{x + 5} - 3 = 0$$
$$\sqrt{x + 5} = 3$$
$$(\sqrt{x + 5})^2 = 3^2$$
$$x + 5 = 9$$
$$x = 4$$

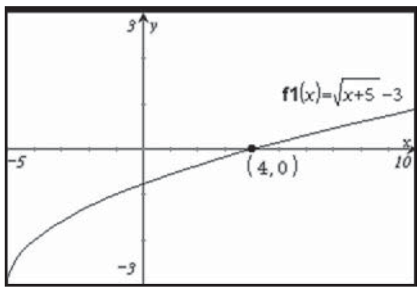
Why do you need to square both sides?

Is this an extraneous root? Does it meet the restrictions on the variable in the square root?

The value $x = 4$ is the root or solution to the equation.

b) To find the x-intercepts of the graph of $y = \sqrt{x + 5} - 3$, graph the function using technology and determine the x-intercepts.

The function has a single x-intercept at (4, 0).



c) The value $x = 4$ is the zero of the function because the value of the function is 0 when $x = 4$. The roots to a radical equation are equal to the x-intercepts of the graph of the corresponding radical function.

Your Turn

a) Use a graph to locate the x-intercept(s) of the graph of $y = \sqrt{x + 2} - 4$.

b) Algebraically determine the root(s) of the equation $\sqrt{x + 2} - 4 = 0$.

c) Describe the relationship between your findings in parts a) and b).

Example 2

Solve a Radical Equation Involving an Extraneous Solution

Solve the equation $\sqrt{x + 5} = x + 3$ algebraically and graphically.

Solution

$$\begin{aligned} \sqrt{x + 5} &= x + 3 \\ (\sqrt{x + 5})^2 &= (x + 3)^2 \\ x + 5 &= x^2 + 6x + 9 \\ 0 &= x^2 + 5x + 4 \\ 0 &= (x + 4)(x + 1) \end{aligned}$$

$x + 4 = 0$ or $x + 1 = 0$

$x = -4$ $\qquad$ $x = -1$

Check:
Substitute $x = -4$ and $x = -1$ into the original equation to identify any extraneous roots.

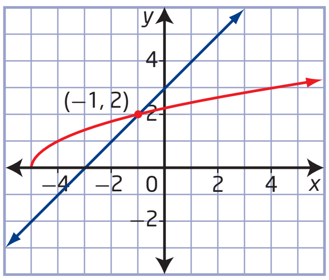
Why do extraneous roots occur?

Left Side	Right Side	Left Side	Right Side
$\sqrt{x + 5}$	$x + 3$	$\sqrt{x + 5}$	$x + 3$
$= \sqrt{-4 + 5}$	$= -4 + 3$	$= \sqrt{-1 + 5}$	$= -1 + 3$
$= \sqrt{1}$	$= -1$	$= \sqrt{4}$	$= 2$
$= 1$		$= 2$	
Left Side ≠ Right Side		Left Side = Right Side	

The solution is $x = -1$.

Solve the equation graphically using functions to represent the two sides of the equation.

$y_1 = \sqrt{x + 5}$
$y_2 = x + 3$



The two functions intersect at the point $(-1, 2)$. The value of x at this point, $x = -1$, is the solution to the equation.

Your Turn

Solve the equation $4 - x = \sqrt{6 - x}$ graphically and algebraically.

Example 3

Approximate Solutions to Radical Equations

a) Solve the equation $\sqrt{3x^2 - 5} = x + 4$ graphically. Express your answer to the nearest tenth.

b) Verify your solution algebraically.

Solution

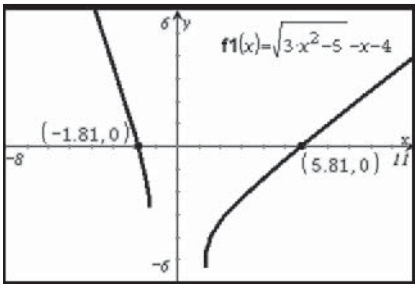
a) To determine the roots or solutions to an equation of the form $f(x) = g(x)$, identify the x-intercepts of the graph of the corresponding function, $y = f(x) - g(x)$.

Method 1: Use a Single Function

Rearrange the radical equation so that one side is equal to zero:

$$\sqrt{3x^2 - 5} = x + 4$$
$$\sqrt{3x^2 - 5} - x - 4 = 0$$

Graph the corresponding function, $y = \sqrt{3x^2 - 5} - x - 4$, and determine the x-intercepts of the graph.



The values of the x-intercepts of the graph are the same as the solutions to the original equation. Therefore, the solution is $x \approx -1.8$ and $x \approx 5.8$.

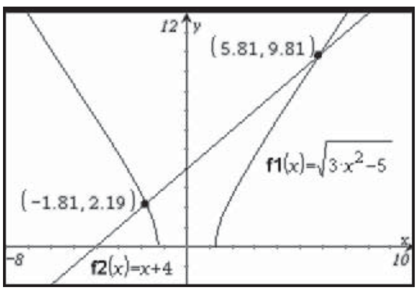
Method 2: Use a System of Two Functions

Express each side of the equation as a function:

$y_1 = \sqrt{3x^2 - 5}$

$y_2 = x + 4$

Graph these functions and determine the value of x at the point(s) of intersection, i.e., where $y_1 = y_2$.



The solution to the equation $\sqrt{3x^2 - 5} = x + 4$ is $x \approx -1.8$ and $x \approx 5.8$.

b) Identify the values of x for which the radical is defined.

$$3x^2 - 5 \geq 0$$
$$3x^2 \geq 5$$
$$x^2 \geq \frac{5}{3}$$
$$|x| \geq \sqrt{\frac{5}{3}}$$

Case 1

If $x \geq 0$, $x \geq \sqrt{\frac{5}{3}}$.

Case 2

If $x < 0$, $x < -\sqrt{\frac{5}{3}}$.

Solve for x:

$$\sqrt{3x^2 - 5} = x + 4$$
$$(\sqrt{3x^2 - 5})^2 = (x + 4)^2$$

Why do you need to square both sides?

$$3x^2 - 5 = x^2 + 8x + 16$$
$$2x^2 - 8x - 21 = 0$$
$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(2)(-21)}}{2(2)}$$

Why does the quadratic formula need to be used here?

$$x = \frac{8 \pm \sqrt{232}}{4}$$
$$x = \frac{8 \pm 2\sqrt{58}}{4}$$
$$x = \frac{4 \pm \sqrt{58}}{2}$$

$x = \frac{4 + \sqrt{58}}{2}$ or $x = \frac{4 - \sqrt{58}}{2}$

$x \approx 5.8$ $\quad$ $x \approx -1.8$

Do these solutions meet the restrictions on x? How can you determine whether either of the roots is extraneous?

The algebraic method gives an exact solution. The approximate solution obtained algebraically, $x \approx -1.8$ and $x \approx 5.8$, is the same as the approximate solution obtained graphically.

Your Turn

Solve the equation $x + 3 = \sqrt{12 - 2x^2}$ using two different methods.

Example 4

Solve a Problem Involving a Radical Equation

An engineer designs a roller coaster that involves a vertical drop section just below the top of the ride. She uses the equation $v = \sqrt{(v_0)^2 + 2ad}$ to model the velocity, v, in feet per second, of the ride's cars after dropping a distance, d, in feet, with an initial velocity, v_0, in feet per second, at the top of the drop, and constant acceleration, a, in feet per second squared. The design specifies that the speed of the ride's cars be 120 ft/s at the bottom of the vertical drop section. If the initial velocity of the coaster at the top of the drop is 10 ft/s and the only acceleration is due to gravity, 32 ft/s², what vertical drop distance should be used, to the nearest foot?

Did You Know?

Top Thrill Dragster is a vertical drop-launched roller coaster in Cedar Point amusement park, in Sandusky, Ohio. When it opened in 2003, it set three new records for roller coasters: tallest, fastest top speed, and steepest drop. It stands almost 130 m tall, and on a clear day riders at the top can see Canada's Pelee Island across Lake Erie.

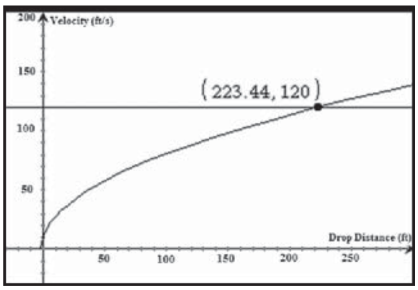
Solution

Substitute the known values into the formula. Then, graph the functions that correspond to both sides of the equation and determine the point of intersection.

$$v = \sqrt{(v_0)^2 + 2ad}$$
$$120 = \sqrt{(10)^2 + 2(32)d}$$
$$120 = \sqrt{100 + 64d}$$

What two functions do you need to graph?

The intersection point indicates that the drop distance should be approximately 223 ft to result in a velocity of 120 ft/s at the bottom of the drop.



Web Link

To see a computer animation of *Top Thrill Dragster*, go to www.mcgrawhill.ca/school/learningcentres and follow the links.

Your Turn

Determine the initial velocity required in a roller coaster design if the velocity will be 26 m/s at the bottom of a vertical drop of 34 m. (Acceleration due to gravity in SI units is 9.8 m/s².)

Key Ideas

- You can solve radical equations algebraically and graphically.
- The solutions or roots of a radical equation are equivalent to the x-intercepts of the graph of the corresponding radical function. You can use either of the following methods to solve radical equations graphically:
  - Graph the corresponding function and identify the value(s) of the x-intercept(s).
  - Graph the system of functions that corresponds to the expression on each side of the equal sign, and then identify the value(s) of x at the point(s) of intersection.

Check Your Understanding

Practise

1. Match each equation to the single function that can be used to solve it graphically. For all equations, $x \geq -4$.

a) $2 + \sqrt{x + 4} = 4$

b) $x - 4 = \sqrt{x + 4}$

c) $2 = \sqrt{x + 4} - 4$

d) $\sqrt{x + 4} + 2 = x + 6$

A $y = x - 4 - \sqrt{x + 4}$

B $y = \sqrt{x + 4} - 2$

C $y = \sqrt{x + 4} - x - 4$

D $y = \sqrt{x + 4} - 6$

2. a) Determine the root(s) of the equation $\sqrt{x + 7} - 4 = 0$ algebraically.

b) Determine the x-intercept(s) of the graph of the function $y = \sqrt{x + 7} - 4$ graphically.

c) Explain the connection between the root(s) of the equation and the x-intercept(s) of the graph of the function.

3. Determine the approximate solution to each equation graphically. Express your answers to three decimal places.

a) $\sqrt{7x - 4} = 13$

b) $9 + \sqrt{6 - 11x} = 45$

c) $\sqrt{x^2 + 2} - 5 = 0$

d) $45 - \sqrt{10 - 2x^2} = 25$

4. a) Solve the equation $2\sqrt{3x + 5} + 7 = 16$, $x \geq -\frac{5}{3}$, algebraically.

b) Show how you can use the graph of the function $y = 2\sqrt{3x + 5} - 9$, $x \geq -\frac{5}{3}$, to find the solution to the equation in part a).

5. Solve each equation graphically. Identify any restrictions on the variable.

a) $\sqrt{2x - 9} = 11$

b) $7 = \sqrt{12 - x} + 4$

c) $5 + 2\sqrt{5x + 32} = 12$

d) $5 = 13 - \sqrt{25 - 2x}$

6. Solve each equation algebraically. What are the restrictions on the variables?

a) $\sqrt{5x^2 + 11} = x + 5$

b) $x + 3 = \sqrt{2x^2 - 7}$

c) $\sqrt{13 - 4x^2} = 2 - x$

d) $x + \sqrt{-2x^2 + 9} = 3$

7. Solve each equation algebraically and graphically. Identify any restrictions on the variables.

a) $\sqrt{8 - x} = x + 6$

b) $4 = x + 2\sqrt{x - 7}$

c) $\sqrt{3x^2 - 11} = x + 1$

d) $x = \sqrt{2x^2 - 8} + 2$

Apply

8. Determine, graphically, the approximate value(s) of a in each formula if $b = 6.2$, $c = 9.7$, and $d = -12.9$. Express answers to the nearest hundredth.

a) $c = \sqrt{ab - d}$

b) $d + 7\sqrt{a + c} = b$

c) $c = b - \sqrt{a^2 + d}$

d) $\sqrt{2a^2 + c} + d = a - b$

9. Naomi says that the equation $6 + \sqrt{x + 4} = 2$ has no solutions.

a) Show that Naomi is correct, using both a graphical and an algebraic approach.

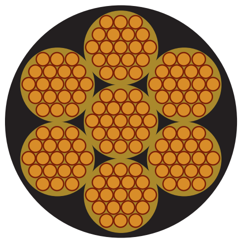
b) Is it possible to tell that this equation has no solutions just by examining the equation? Explain.

10. Two researchers, Greg and Yolanda, use the function $N(t) = 1.3\sqrt{t} + 4.2$ to model the number of people that might be affected by a certain medical condition in a region of 7.4 million people. In the function, N represents the number of people, in millions, affected after t years. Greg predicts that the entire population would be affected after 6 years. Yolanda believes that it would take only 1.5 years. Who is correct? Justify your answer.

11. The period, T, in seconds, of a pendulum depends on the distance, L, in metres, between the pivot and the pendulum's centre of mass. If the initial swing angle is relatively small, the period is given by the radical function $T = 2\pi\sqrt{\frac{L}{g}}$, where g represents acceleration due to gravity (approximately 9.8 m/s² on Earth). Jeremy is building a machine and needs it to have a pendulum that takes 1 s to swing from one side to the other. How long should the pendulum be, in centimetres?

12. Cables and ropes are made of several strands that contain individual wires or threads. The term "7 × 19 cable" refers to a cable with 7 strands, each containing 19 wires.

Suppose a manufacturer uses the function $d = \sqrt{\frac{b}{30}}$ to relate the diameter, d, in millimetres, of its 7 × 19 stainless steel aircraft cable to the safe working load, b, in kilograms.



a) Is a cable with a diameter of 6.4 mm large enough to support a mass of 1000 kg?

b) What is the safe working load for a cable that is 10 mm in diameter?

Did You Know?

The safe working load for a cable or rope is related to its breaking strength, or minimum mass required for it to break. To ensure safety, manufacturers rate a cable's safe working load to be much less than its actual breaking strength.

13. Hazeem states that the equations $\sqrt{x^2} = 9$ and $(\sqrt{x})^2 = 9$ have the same solution. Is he correct? Justify your answer.

14. What real number is exactly one greater than its square root?

15. A parachute-manufacturing company uses the formula $d = 3.69\sqrt{\frac{m}{v^2}}$ to model the diameter, d, in metres, of its dome-shaped circular parachutes so that an object with mass, m, in kilograms, has a descent velocity, v, in metres per second, under the parachute.

a) What is the landing velocity for a 20-kg object using a parachute that is 3.2 m in diameter? Express your answer to the nearest metre per second.

b) A velocity of 2 m/s is considered safe for a parachutist to land. If the parachute has a diameter of 16 m, what is the maximum mass of the parachutist, in kilograms?

Extend

16. If the function $y = \sqrt{-3(x + c)} + c$ passes through the point $(-1, 1)$, what is the value of c? Confirm your answer graphically, and use the graph to create a similar question for the same function.

17. Heron's formula, $A = \sqrt{s(s - a)(s - b)(s - c)}$, relates the area, A, of a triangle to the lengths of the three sides, a, b, and c, and its semi-perimeter (half its perimeter), $s = \frac{a + b + c}{2}$. A triangle has an area of 900 cm^2 and one side that measures 60 cm. The other two side lengths are unknown, but one is twice the length of the other. What are the lengths of the three sides of the triangle?

Create Connections

C1 How can the graph of a function be used to find the solutions to an equation? Create an example to support your answer.

C2 The speed, in metres per second, of a tsunami travelling across the ocean is equal to the square root of the product of the depth of the water, in metres, and the acceleration due to gravity, 9.8 m/s^2.

a) Write a function for the speed of a tsunami. Define the variables you used.

b) Calculate the speed of a wave at a depth of 2500 m, and use unit analysis to show that the resulting speed has the correct units.

c) What depth of water would produce a speed of 200 m/s? Solve graphically and algebraically.

d) Which method of solving do you prefer in this case: algebraic or graphical? Do you always prefer one method over the other, or does it depend? Explain.

C3 Does every radical equation have at least one solution? How can using a graphical approach to solving equations help you answer this question? Support your answer with at least two examples.

C4 Describe two methods of identifying extraneous roots in a solution to a radical equation. Explain why extraneous roots may occur.

Chapter 2 Review

2.1 Radical Functions and Transformations, pages 62–77

1. Graph each function. Identify the domain and range, and explain how they connect to the values in a table of values and the shape of the graph.

a) $y = \sqrt{x}$

b) $y = \sqrt{3 - x}$

c) $y = \sqrt{2x + 7}$

2. What transformations can you apply to $y = \sqrt{x}$ to obtain the graph of each function? State the domain and range in each case.

a) $y = 5\sqrt{x + 20}$

b) $y = \sqrt{-2x} - 8$

c) $y = -\sqrt{\frac{1}{6}(x - 11)}$

3. Write the equation and state the domain and range of the radical function that results from each set of transformations on the graph of $y = \sqrt{x}$.

a) a horizontal stretch by a factor of 10 and a vertical translation of 12 units up

b) a vertical stretch by a factor of 2.5, a reflection in the x-axis, and a horizontal translation of 9 units left

c) a horizontal stretch by a factor of $\frac{5}{2}$, a vertical stretch by a factor of $\frac{1}{20}$, a reflection in the y-axis, and a translation of 7 units right and 3 units down

4. Sketch the graph of each function by transforming the graph of $y = \sqrt{x}$. State the domain and range of each.

a) $y = -\sqrt{x - 1} + 2$

b) $y = 3\sqrt{-x} - 4$

c) $y = \sqrt{2(x + 3)} + 1$

5. How can you use transformations to identify the domain and range of the function $y = -2\sqrt{3(x - 4)} + 9$?

6. The sales, S, in units, of a new product can be modelled as a function of the time, t, in days, since it first appears in stores using the function $S(t) = 500 + 100\sqrt{t}$.

a) Describe how to graph the function by transforming the graph of $y = \sqrt{t}$.

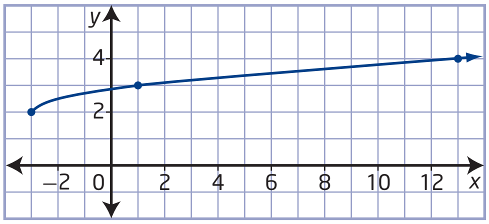
b) Graph the function and explain what the shape of the graph indicates about the situation.

c) What are the domain and range? What do they mean in this situation?

d) Predict the number of items sold after 60 days.

7. Write an equation of the form $y = a\sqrt{b(x - h)} + k$ for each graph.

a)



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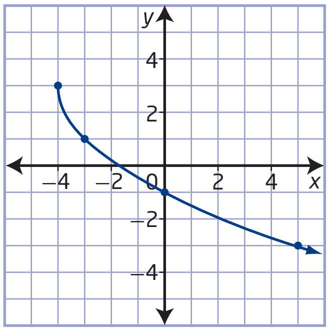

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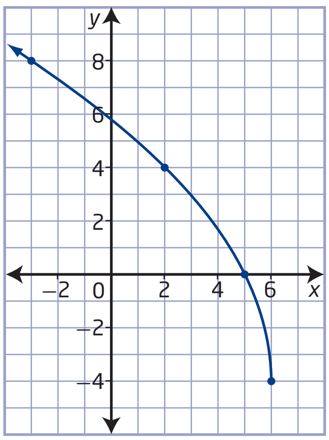


2.2 Square Root of a Function, pages 78–89

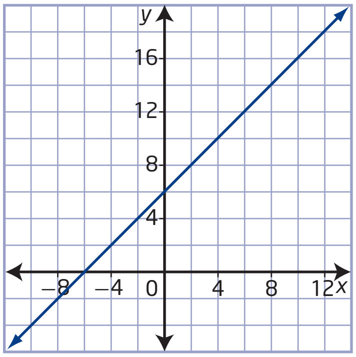
8. Identify the domains and ranges of the functions in each pair and explain any differences.

a) $y = x - 2$ and $y = \sqrt{x - 2}$

b) $y = 10 - x$ and $y = \sqrt{10 - x}$

c) $y = 4x + 11$ and $y = \sqrt{4x + 11}$

9. The graph of $y = f(x)$ is shown.



a) Graph the function $y = \sqrt{f(x)}$ and describe your strategy.

b) Explain how the graphs are related.

c) Identify the domain and range of each function and explain any differences.

10. Identify and compare the domains and ranges of the functions in each pair, and explain why they differ.

a) $y = 4 - x^2$ and $y = \sqrt{4 - x^2}$

b) $y = 2x^2 + 24$ and $y = \sqrt{2x^2 + 24}$

c) $y = x^2 - 6x$ and $y = \sqrt{x^2 - 6x}$

11. A 25-ft-long ladder leans against a wall. The height, h, in feet, of the top of the ladder above the ground is related to its distance, d, in feet, from the base of the wall.

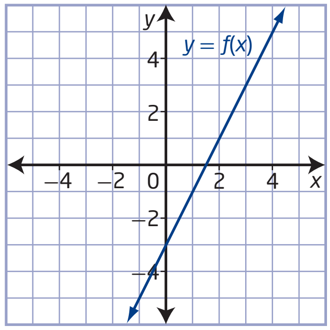
a) Write an equation to represent h as a function of d.

b) Graph the function and identify the domain and range.

c) Explain how the shape of the graph, the domain, and the range relate to the situation.

12. Using each graph of $y = f(x)$, sketch the graph of $y = \sqrt{f(x)}$.

a)



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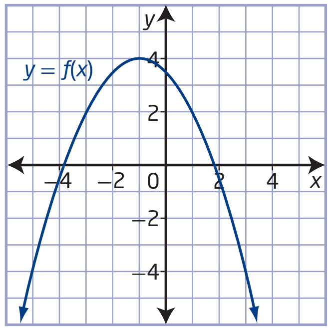

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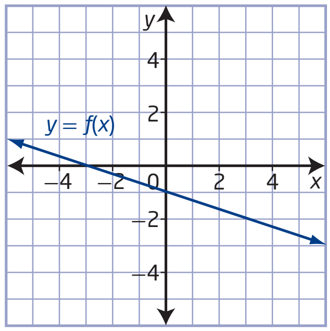


2.3 Solving Radical Equations Graphically, pages 90–98

13. a) Determine the root(s) of the equation $\sqrt{x + 3} - 7 = 0$ algebraically.

b) Use a graph to locate the x-intercept(s) of the function $f(x) = \sqrt{x + 3} - 7$.

c) Use your answers to describe the connection between the x-intercepts of the graph of a function and the roots of the corresponding equation.

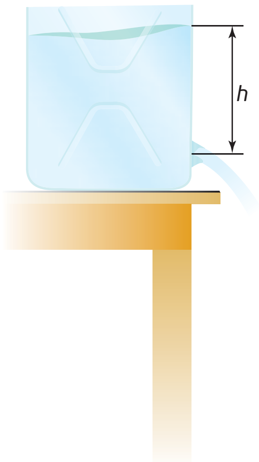
14. Determine the approximate solution to each equation graphically. Express answers to three decimal places.

a) $\sqrt{7x - 9} - 4 = 0$

b) $50 = 12 + \sqrt{8 - 12x}$

c) $\sqrt{2x^2 + 5} = 11$

15. The speed, s, in metres per second, of water flowing out of a hole near the bottom of a tank relates to the height, h, in metres, of the water above the hole by the formula $s = \sqrt{2gh}$. In the formula, g represents the acceleration due to gravity, 9.8 m/s^2. At what height is the water flowing out a speed of 9 m/s?



Did You Know?

The speed of fluid flowing out of a hole near the bottom of a tank filled to a depth, h, is the same as the speed an object acquires in falling freely from the height h. This relationship was discovered by Italian scientist Evangelista Torricelli in 1643 and is referred to as Torricelli's law.

16. Solve each equation graphically and algebraically.

a) $\sqrt{5x + 14} = 9$

b) $7 + \sqrt{8 - x} = 12$

c) $23 - 4\sqrt{2x - 10} = 12$

d) $x + 3 = \sqrt{18 - 2x^2}$

17. Atid, Carly, and Jaime use different methods to solve the radical equation $3 + \sqrt{x - 1} = x$.

Their solutions are as follows:

- Atid: $x = 2$
- Carly: $x = 5$
- Jaime: $x = 2, 5$

a) Who used an algebraic approach? Justify your answer.

b) Who used a graphical method? How do you know?

c) Who made an error in solving the equation? Justify your answer.

18. Assume that the shape of a tipi approximates a cone. The surface area, S, in square metres, of the walls of a tipi can be modelled by the function $S(r) = \pi r\sqrt{36 + r^2}$, where r represents the radius of the base, in metres.

Blackfoot Crossing, Alberta

a) If a tipi has a radius of 5.2 m, what is the minimum area of canvas required for the walls, to the nearest square metre?

b) If you use 160 m^2 of canvas to make the walls for this tipi, what radius will you use?

Chapter 2 Practice Test

Multiple Choice

For #1 to #6, choose the best answer.

1. If $f(x) = x + 1$, which point is on the graph of $y = \sqrt{f(x)}$?

A (0, 0) **B** (0, 1)

C (1, 0) **D** (1, 1)

2. Which intercepts will help you find the roots of the equation $\sqrt{2x - 5} = 4$?

A x-intercepts of the graph of the function $y = \sqrt{2x - 5} - 4$

B x-intercepts of the graph of the function $y = \sqrt{2x - 5} + 4$

C y-intercepts of the graph of the function $y = \sqrt{2x - 5} - 4$

D y-intercepts of the graph of the function $y = \sqrt{2x - 5} + 4$

3. Which function has a domain of $\{x \mid x \geq 5, x \in \mathrm{R}\}$ and a range of $\{y \mid y \geq 0, y \in \mathrm{R}\}$?

A $f(x) = \sqrt{x - 5}$

B $f(x) = \sqrt{x} - 5$

C $f(x) = \sqrt{x + 5}$

D $f(x) = \sqrt{x} + 5$

4. If $y = \sqrt{x}$ is stretched horizontally by a factor of 6, which function results?

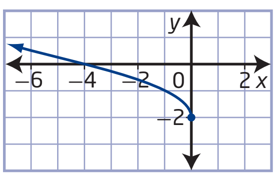
A $y = \frac{1}{6}\sqrt{x}$

B $y = 6\sqrt{x}$

C $y = \sqrt{\frac{1}{6}x}$

D $y = \sqrt{6x}$

5. Which equation represents the function shown in the graph?



A $y - 2 = -\sqrt{x}$ **B** $y + 2 = -\sqrt{x}$

C $y - 2 = \sqrt{-x}$ **D** $y + 2 = \sqrt{-x}$

6. How do the domains and ranges compare for the functions $y = \sqrt{x}$ and $y = \sqrt{5x} + 8$?

A Only the domains differ.

B Only the ranges differ.

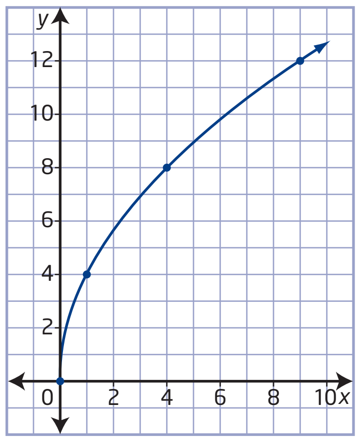
C Both the domains and ranges differ.

D Neither the domains nor the ranges differ.

Short Answer

7. Solve the equation $5 + \sqrt{9 - 13x} = 20$ graphically. Express your answer to the nearest hundredth.

8. Determine two forms of the equation that represents the function shown in the graph.



9. How are the domains and ranges of the functions $y = 7 - x$ and $y = \sqrt{7 - x}$ related? Explain why they differ.

10. If $f(x) = 8 - 2x^2$, what are the domains and ranges of $y = f(x)$ and $y = \sqrt{f(x)}$?

11. Solve the equation $\sqrt{12 - 3x^2} = x + 2$ using two different graphical methods. Show your graphs.

12. Solve the equation $4 + \sqrt{x + 1} = x$ graphically and algebraically. Express your answer to the nearest tenth.

13. The radical function $S = \sqrt{255d}$ can be used to estimate the speed, S, in kilometres per hour, of a vehicle before it brakes from the length, d, in metres, of the skid mark. The vehicle has all four wheels braking and skids to a complete stop on a dry road.

a) Use the language of transformations to describe how to create a graph of this function from a graph of the base square root function.

b) Sketch the graph of the function and use it to determine the approximate length of skid mark expected from a vehicle travelling at 100 km/h on this road.

Extended Response

14. **a)** How can you use transformations to graph the function $y = -\sqrt{2x} + 3$?

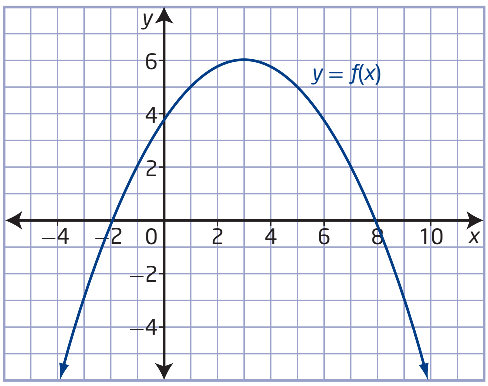
b) Sketch the graph.

c) Identify the domain and range of the function.

d) Describe how the domain and range connect to your answer to part a).

e) How can the graph be used to solve the equation $5 + \sqrt{2x} = 8$?

15. Using the graph of $y = f(x)$, sketch the graph of $y = \sqrt{f(x)}$ and explain your strategy.



16. Consider the roof of the mosque at the Canadian Islamic Centre in Edmonton, Alberta. The diameter of the base of the roof is approximately 10 m, and the vertical distance from the centre of the roof to the base is approximately 5 m.

Canadian Islamic Centre (Al-Rashid), Edmonton, Alberta

a) Determine a function of the form $y = a\sqrt{b(x - h)} + k$, where y represents the distance from the base to the roof and x represents the horizontal distance from the centre.

b) What are the domain and range of this function? How do they relate to the situation?

c) Use the function you wrote in part a) to determine, graphically, the approximate height of the roof at a point 2 m horizontally from the centre of the roof.